CH:23 Electric Field

charge (q): الشحنة الكهربائية (C)

$q = ne$ (الجسم)

- $\rightarrow 1.6 * 10^{-19}$ C $\Leftarrow$ الجسم يفقد $\underline{e}$ (lose)
- $\rightarrow -1.6 * 10^{-19}$ C $\Leftarrow$ الجسم يكتسب $\underline{e}$ (accept)

n: عدد الـ $\underline{e}$ المكتسبة أو المفقودة وهي عدد صحيح موجب

Ex: If an object accepts $2*10^{7}$ e, what is its charge?

$$q = ne$$
$$= 2*10^{7} * -1.6*10^{-19}$$
$$= -3.2 * 10^{-12}\ C.$$

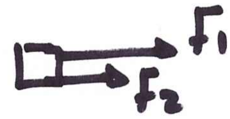
Ex: If an object contains $4*10^{3}\ e^{-}$ and $2*10^{3}\ p^{+}$ what is its net charge?

$$q = ne + np$$
(الجسم)
$$= 4*10^{3} * -1.6*10^{-19} + 2*10^{3} * 1.6*10^{-19}$$
$$= -6.4*10^{-16} + 3.2*10^{-16}$$
$$= -3.2*10^{-16}\ C.$$

٢) محصلة المتجهات :-

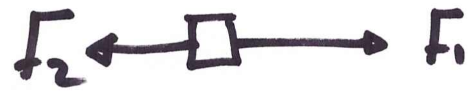
١) إذا كان المتجهين بنفس الاتجاه :

$F_{net} = F_1 + F_2$



٢) إذا كانت الكميتان المتجهتان متعاكستان :

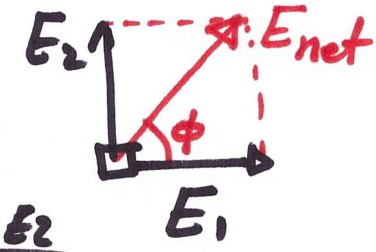
$\boxed{F_{net} = F_1 - F_2}$ باتجاه الأكبر



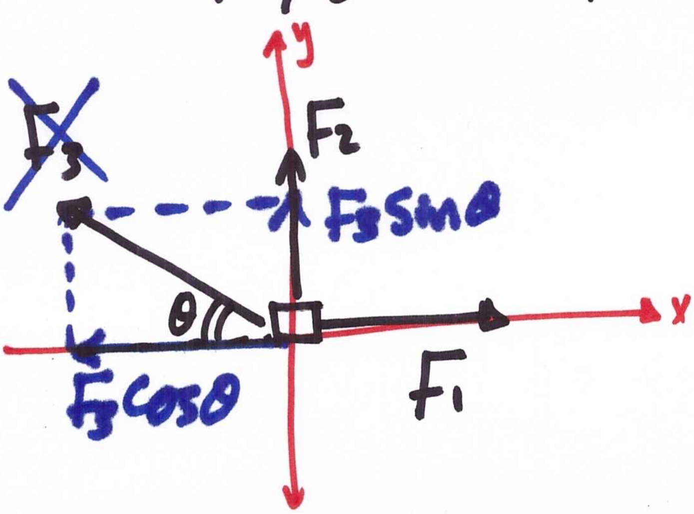
٣) إذا كان المتجهان متعامدان :

$E_{net} = \sqrt{E_1^2 + E_2^2}$

$\tan\phi = \frac{E_2}{E_1} \Rightarrow \phi = \tan^{-1}\frac{E_2}{E_1}$



٤) إذا أثر على الجسم أكثر من متجهين ، أو إذا كانت $\theta \neq 90°$



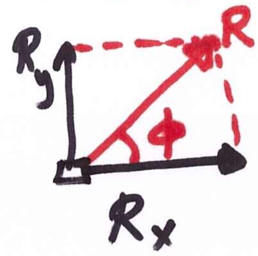
١) تحديد المحاور :

٢) تحليل أي قوة لا تنطبق على المحاور

٣) نجد المحصلة السينية والصادية :

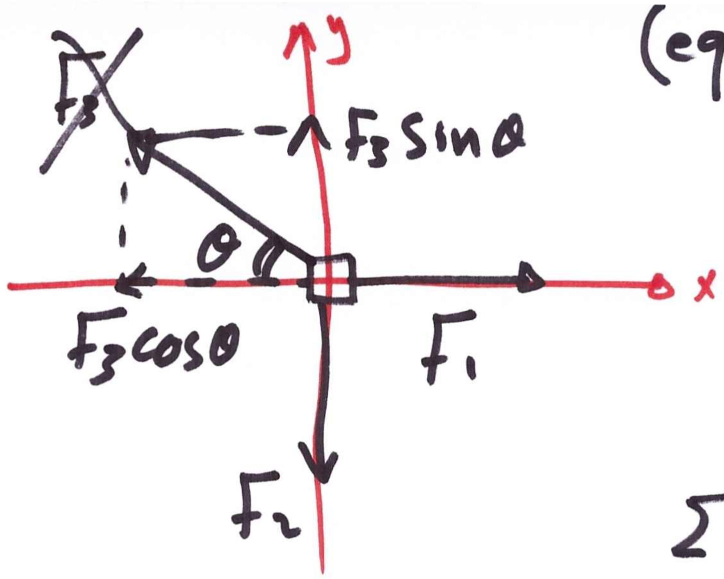
$R_x = F_1 - F_3\cos\theta$

$R_y = F_2 + F_3\sin\theta$



٤) نجد R : $R = \sqrt{R_x^2 + R_y^2}$, $\tan\phi = \frac{R_y}{R_x}$

3)



٥ الجسم متزن (equilibrium)

١) تحديد المحاور

٢) نحلل القوة الغير منطبقة على المحاور.

٣) نطبق قانوني الإتزان :-

$$\sum F_{\rightarrow} = \sum F_{\leftarrow} \quad , \quad \sum F_{\uparrow} = \sum F_{\downarrow}$$

$$F_1 = F_3 \cos\theta \qquad \Big\| \qquad F_3 \sin\theta = F_2$$

Coulomb's law

$$F_e = \frac{k\, q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 9*10^{9}$$

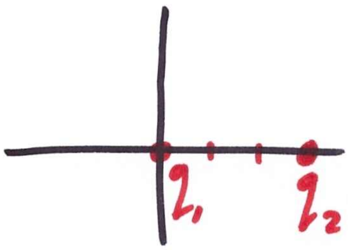
$$\epsilon_0 = 8.85*10^{-12}$$

* الشحنة سالبة ، فإننا <u>لا</u> نعوض الإشارة السالبة في قانون كولوم

$K = 10^3$, $M = 10^6$, $G = 10^9$

$c = 10^{-2}$, $m = 10^{-3}$, $\mu = 10^{-6}$, $n = 10^{-9}$, $P = 10^{-12}$, $f = 10^{-15}$

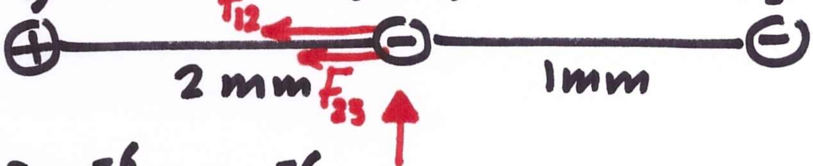
Ex: $q_1 = 8\mu c$ $\quad q_2 = -6\mu c$



$$F = \frac{9*10^{9} * 8*10^{-6} * 6*10^{-6}}{(3*10^{-2})^2} = 48*10 = 480\,N \text{ (attraction)}$$

Ex: $q_1 = 9\mu c$ $\quad q_2 = -4\mu c$ $\quad q_3 = -1\mu c$

What is the <u>net</u> force that acting on <u>q_2</u>.



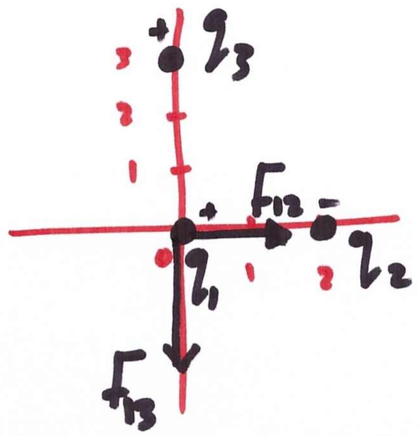
$$F_{12} = \frac{9\times10^{9} * 9*10^{-6} * 4*10^{-6}}{(2*10^{-3})^2} = 81*10^{3}\,N$$

$$F_{23} = \frac{9*10^{9} * 4*10^{-6} * 1*10^{-6}}{(1*10^{-3})^2} = 36*10^{3}\,N$$

$$F_{net} = F_{12} + F_{23} = 117*10^{3}\,N\,(-\hat{i})$$

* اتجاهات
* حسابات
* محصلة.

Ex: If we have 3-charges, $q_1 = 1nC$ at origin and $q_2 = -4nC$ at $x = 2cm$, while $q_3 = 12nC$ at $y = 3cm$, Find the force acting on q_1.



$$F_{12} = \frac{9\times10^{9}\times1\times10^{-9}\times4\times10^{-9}}{(2\times10^{-2})^2}$$

$$= 9\times10^{-5}N$$

$$F_{13} = \frac{9\times10^{9}\times1\times10^{-9}\times12\times10^{-9}}{(3\times10^{-2})^2}$$

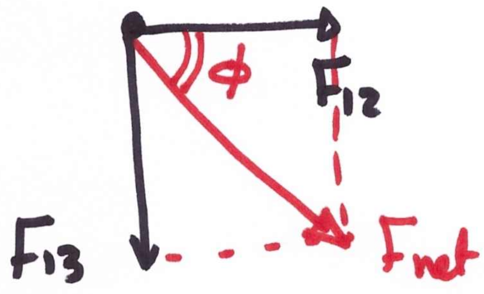
$$= 12\times10^{-5}N.$$

$$F_{net} = \sqrt{F_{12}^2 + F_{13}^2}$$

$$= 10^{-5}\sqrt{9^2 + 12^2}$$

$$= 10^{-5}\sqrt{81+144}$$

$$= 15\times10^{-5}N$$

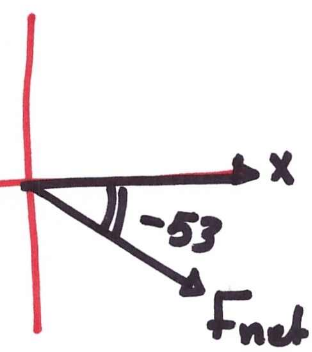


$$\tan\phi = \frac{12\times\cancel{10^{-5}}}{9\times\cancel{10^{-5}}}$$

$$= \frac{4}{3}$$

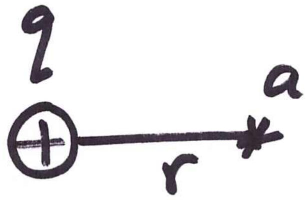
$$\Rightarrow \boxed{\phi = 53.1^\circ}$$

direction of F_{net} is $\boxed{-53^\circ}$ or 307°.

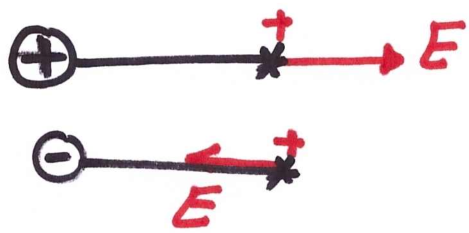


Electric Field

$$\boxed{E = \frac{k\,q}{r^2}}$$

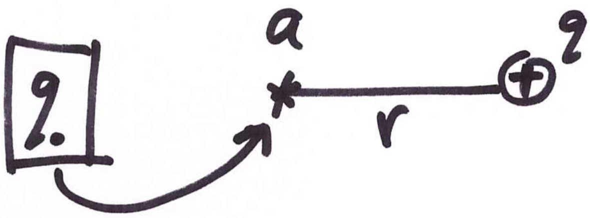


* الإشارة السالبة لا تعوض .



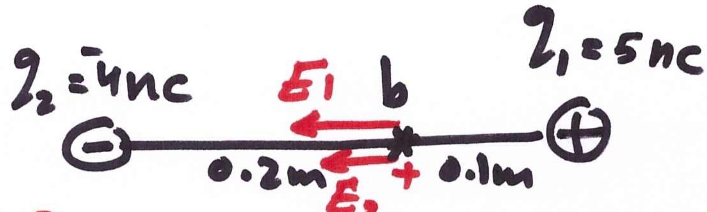
F مع اتجاه E
$q_0 = \oplus$

F عكس اتجاه E
$q_0 = \ominus$



$$\Rightarrow F = q_0 E_a$$

Ex: calculate the $\underline{E}$ at $\underline{b}$.



$$E_1 = \frac{9\times10^9 * 5*10^{-9}}{(1*10^{-1})^2} = 45*10^2\, N/C$$

$$E_2 = \frac{9\times10^9 * 4*10^{-9}}{(2*10^{-1})^2} = 9*10^2\, N/C$$

* اتجاهات .
* حسابات .
* محصلة .

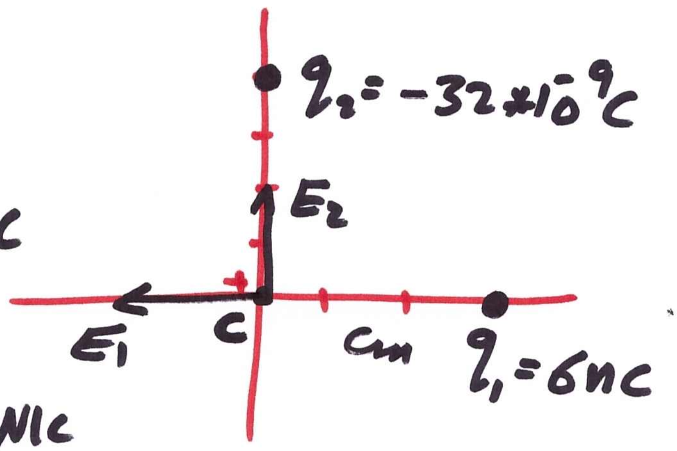
$$E_{net} \equiv E_b = E_1 + E_2 = 54*10^2\, N/C\ (-i)$$

$$F = q_0 E_b = 2*10^{-3} * 45*10^2 = 90*10^{-1} = 9N$$

b ← q_0 *

Ex: Calculate the Electric Field at **C**.

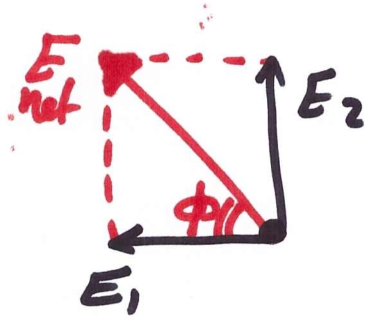
Sol:



$$E_1 = \frac{9\times10^{9} \times 6\times10^{-9}}{(3\times10^{-2})^2} = 6\times10^{4}\ N/C$$

$$E_2 = \frac{9\times10^{9} \times 32\times10^{-9}}{(4\times10^{-2})^2} = 18\times10^{4}\ N/C$$

$$E_{net} = \sqrt{E_1^2 + E_2^2}$$
$$= \sqrt{6^2 + 18^2} \times 10^4$$
$$= 19\times10^{4}\ N/C.$$

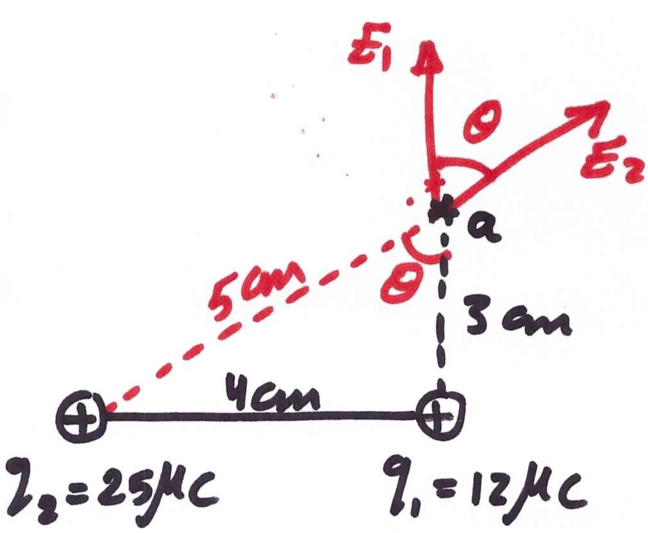


$$\tan\phi = \frac{18\times10^{4}}{6\times10^{4}} = \cancel{4.5}\ 3 \Rightarrow \boxed{\phi = 71.5^\circ}$$

Ex: What is the Electric field at Point a:

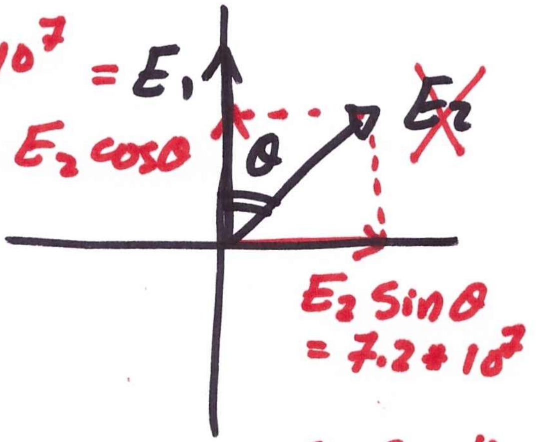
$$E_1 = \frac{9\times10^{9} \times 12\times10^{-6}}{9\times10^{-4}} = 12\times10^{7}\ N/C$$

$$E_2 = \frac{9\times10^{9} \times 25\times10^{-6}}{(5\times10^{-2})^2} = 9\times10^{7}\ N/C$$



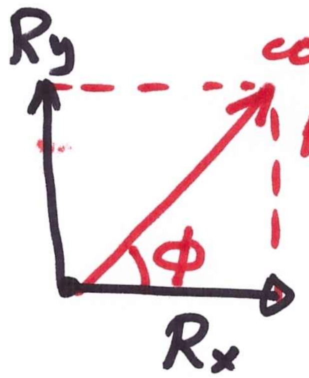
$R_x = 7.2 * 10^7 \ (+\hat{i})$

$R_y = 12*10^7 + 5.4*10^7$
$= 17.4 * 10^7 \ (+\hat{j})$



Sinθ = 4/5 = 0.8
cosθ = 3/5 = 0.6

$R = \sqrt{7.2^2 + 17.4^2} * 10^7$
$= 18.8 * 10^7 \ N/C$



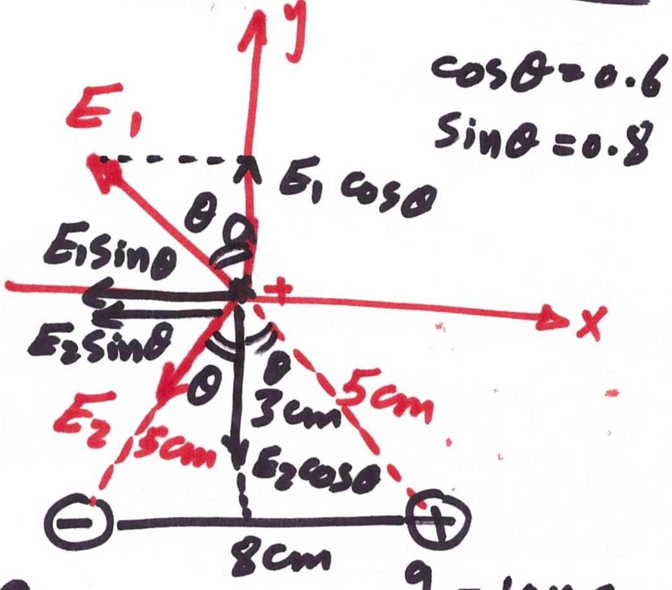
$\tan\phi = \dfrac{17.4*10^7}{7.2*10^7} = 2.4$

$\Rightarrow \phi \simeq 67.4$

Ex: Find E_{net} at a:

$E_1 = \dfrac{9\times10^9 * 10\times10^{-9}}{25*10^{-4}} = 3.6*10^4 \ N/C$

$E_2 = \dfrac{9\times10^9\times10\times10^{-9}}{25\times10^{-4}} = 3.6*10^4 \ N/C$



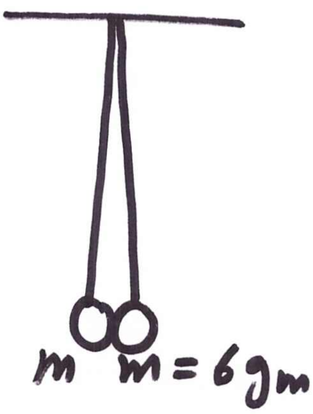
cosθ = 0.6
Sinθ = 0.8

$R_x = E_1 \sin\theta + E_2 \sin\theta$
$= 3.6\times10^4 * 0.8 + 3.6\times10^4 * 0.8 = 2.88*10^4 \ N/C \ (-\hat{i})$

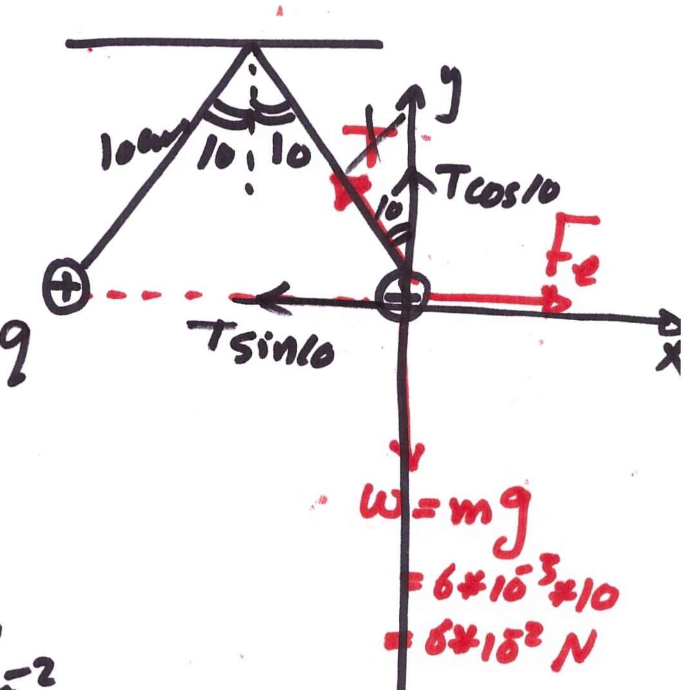
$R_y = E_1\cos\theta - E_2\cos\theta = 0$

$\Rightarrow R = R_x = 2.88*10^4 \ N/C \ \boxed{-\hat{i}}$.

Ex Find q if the system is at equilibrium.



$\Rightarrow$



$$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$$

$$T\cos 10 = 6 * 10^{-2}$$

$$\boxed{T = 0.0609 \text{ New}}$$

$$\Sigma F_{\rightarrow} = \Sigma F_{\leftarrow}$$

$$F_e = T \sin 10$$

$$= 0.0609 * \sin 10$$

$$\boxed{F_e = 1.06 * 10^{-2} \text{ New}}$$

$$F_e = \frac{9*10^{9}\, q_1 q_2}{r^2}$$

$$1.06*10^{-2} = \frac{9\times10^{9} * q^2}{(3.4*10^{-2})^2}$$

$$q^2 = 1.36 * 10^{-7}$$

$$\boxed{q = 3.7 * 10^{-4}\ C}$$

$r = 2x$

$$\sin 10 = \frac{x}{10*10^{-2}}$$

$$\Rightarrow x = 1.7*10^{-2}\,m$$

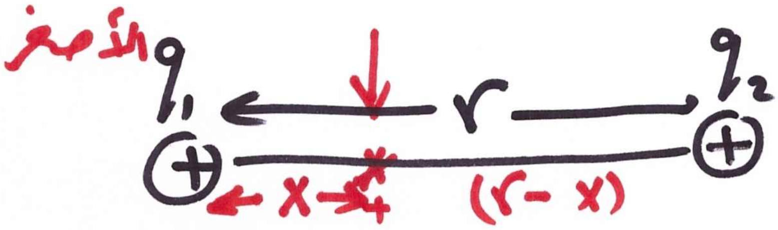
$$\Rightarrow r = 2x = 3.4*10^{-2}\,m$$

equilibrium Point

* هي النقطة التي ينعدم عندها المجال الكهربائي والقوة الكهربائية . ($E = 0$)

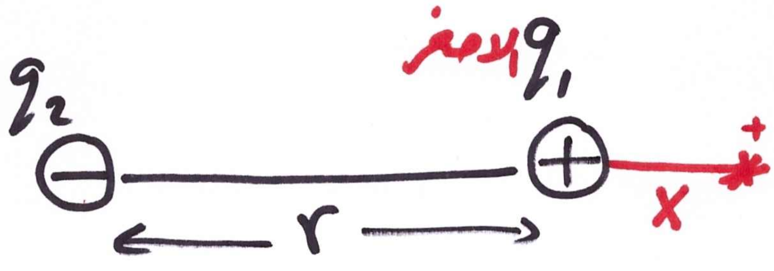
- شحنتان متشابهتان : نقطة التعادل تقع بينهما وأقرب للشحنة الأصغر.

- شحنتان مختلفتان : نقطة التعادل تقع خارجهما أقرب للأصغر



قانون نقطة التعادل ⇐ $\boxed{E_1 = E_2}$

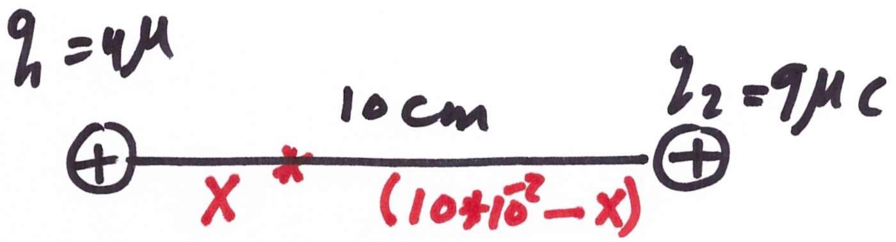
$$\frac{k q_1}{r_1^2} = \frac{k q_2}{r_2^2}$$



Ex: If we two Point charges:

$q_1 = 4\mu C$, $q_2 = 9\mu C$, $r = 10\,cm$

where is the equilibrium Point.
(where is the Point that has <u>no</u> net electric field)



$$E_1 = E_2$$

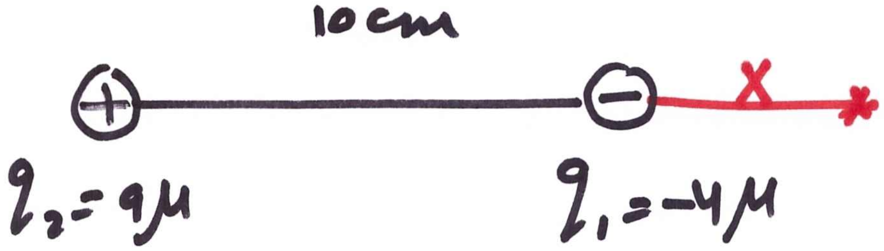
$$\frac{\cancel{K}\ 4*1\cancel{0^{-6}}}{X^2} = \frac{\cancel{K}\ 9*1\cancel{0^{-6}}}{(10*10^{-2} - X)^2}$$

$$\frac{2}{X} = \frac{3}{10\times10^{-2} - X} \Longrightarrow 3X = 20*10^{-2} - 2X$$

$$5X = 20*10^{-2}$$

$$\boxed{X = 4*10^{-2}\,m = 4\,cm}$$

Ex: $q_1 = -4\mu C$, $q_2 = 9\mu C$, $r = 10cm$



$E_1 = E_2$

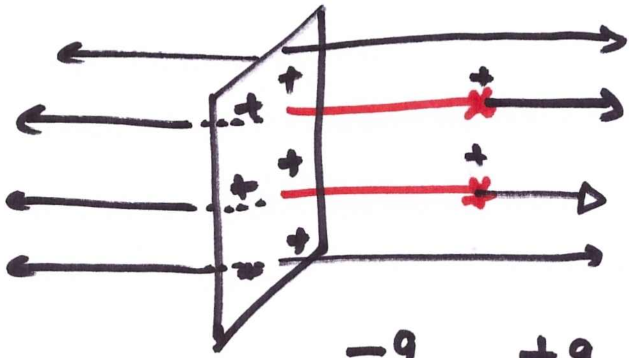
الجذر

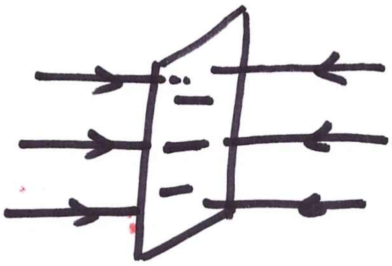
$$\frac{\cancel{9\times10^9}\times 4\times\cancel{10^{-6}}}{X^2} = \frac{\cancel{9\times10^9}\times 9\times\cancel{10^{-6}}}{(10\times10^{-2}+X)^2}$$

$$\frac{2}{X} = \frac{3}{10\times10^{-2}+X} \Longrightarrow 3X = 20\times10^{-2} + 2X$$

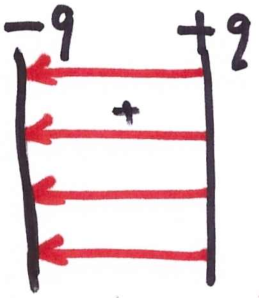
$$\boxed{X = 20\times10^{-2}m = 20\,Cm}$$

Uniform Electric Field





$E = \text{Constant}$

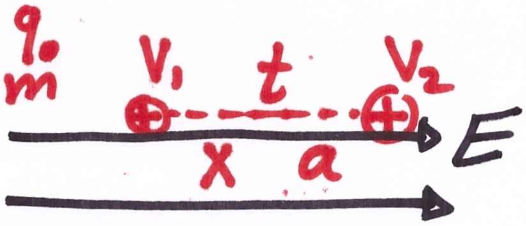


$$\cancel{E_{uni} = \frac{9\times10^{9} \times q}{r^2}}$$

تطبيق 1

حركة جسم مشحون داخل هذا المجال:

Motion of a small charged particle inside the uniform electric field.



$$F_E = q_0 E = ma$$

$$W = F x \cos\theta_{Fx}$$

$$\boxed{\begin{aligned}\Delta K &= K_f - K_i \\ &= \tfrac{1}{2} m V_f^2 - \tfrac{1}{2} m V_i^2\end{aligned}}$$

$$W_{total} = \Delta K$$

$$V_2 = V_1 + at$$

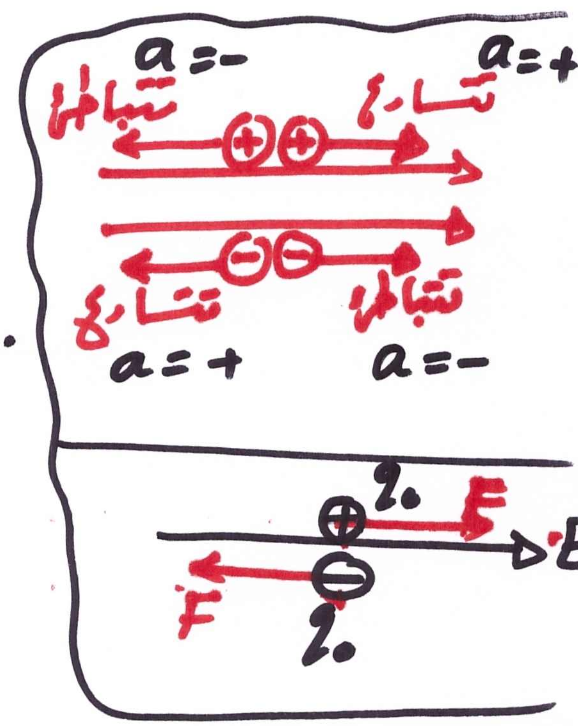
$$V_2^2 = V_1^2 + 2ax$$

$$x = V_1 t + \tfrac{1}{2} a t^2$$

$$x = \left(\frac{V_1 + V_2}{2}\right) t$$

Ex: An object of mass $\underbrace{2*10^{-4}}_{m}$ kgm and charge of $\underbrace{6*10^{-8}\,C}_{q_0}$, enters ~~as~~ a Uniform electric field with speed of $\underbrace{2*10^{4}\,m/s}_{V_1}$ for $\underbrace{10^{3}\,sec}_{t}$. if $\underbrace{E = 4000}_{E}$ N/C as in figure, Find:

1) acceleration
2) Final speed
3) traveled displacement.
4) Force ~~done~~ exerted.
5) Work done.
6) change in Kinetic energy.



Sol:

1) $q_0 E = ma$

$$6*10^{-8} * 4*10^{3} = 2*10^{-4}\,a$$

$$a = 12 * 10^{-1} = {}^{+}1.2\ m/s^2.$$

2) $V_2 = V_1 + at$

$$= 2*10^{4} + 1.2*10^{3}$$

$$= 20000 + 1200$$

$$\boxed{V_2 = 21200\ m/s.}$$

3) $X = V_1 t + \frac{1}{2}at^2$

$$= 2*10^{4}*10^{3} + \frac{1}{2}*1.2*10^{6}$$

$$= 2*10^{7} + 0.6*10^{6}$$

$$\boxed{X = 20.6*10^{6}\ m}$$

4) $F = q_o E = ma$

$= 6*10^{-8} * 4*10^{3}$

$= 24*10^{-5}\ N$

$= 2*10^{-4} * 1.2$

$= 2.4 * 10^{-4}$

$= 24*10^{-5}\ N$

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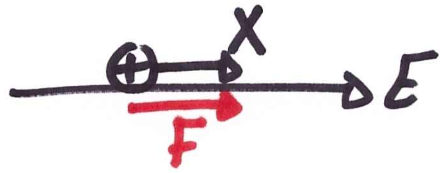
5) $W = F\ X \cos\theta_{Fx}$



$= 24*10^{-5} * 20.6*10^{6} \cos 0$

$= 4944\ J$

---

6) $\Delta K = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2$

$= \frac{1}{\cancel{2}} \cancel{2} * 10^{-4} * (21200)^2 - \frac{1}{\cancel{2}} * \cancel{2} * 10^{-4} * (2*10^{4})^2$

$= 44944 - 40000$

$= 4944\ J$

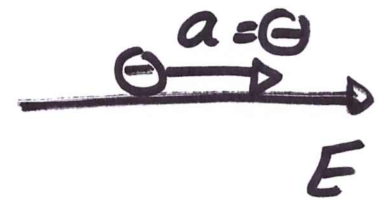
Ex: If an object of mass $\underbrace{0.1\ gm}_{m}$ and charge $\underbrace{-2*10^{-6}\ C}_{q_o}$, start moving with speed of $\underbrace{20 m/s}_{V_1}$ inside a uniform electric field and with the field direction, the distance traveled is $\underbrace{200\ m}_{x}$ Untill the particle was $\underbrace{\text{stopped}}_{V_2 = 0}$, what is the mag. of $\underbrace{\text{Electric field}}_{E}$.

Sol :

$$V_2^2 = V_1^2 + 2ax$$

$$0 = 400 + 2 * a * 200$$

$$a = \frac{-400}{400} = -1\, m/s^2$$



$$q_0 E = ma$$

$$E = \frac{ma}{q_0} = \frac{0.1 * 10^{-3} * 1}{2 * 10^{-6}} = 0.5 * 10^{2}$$

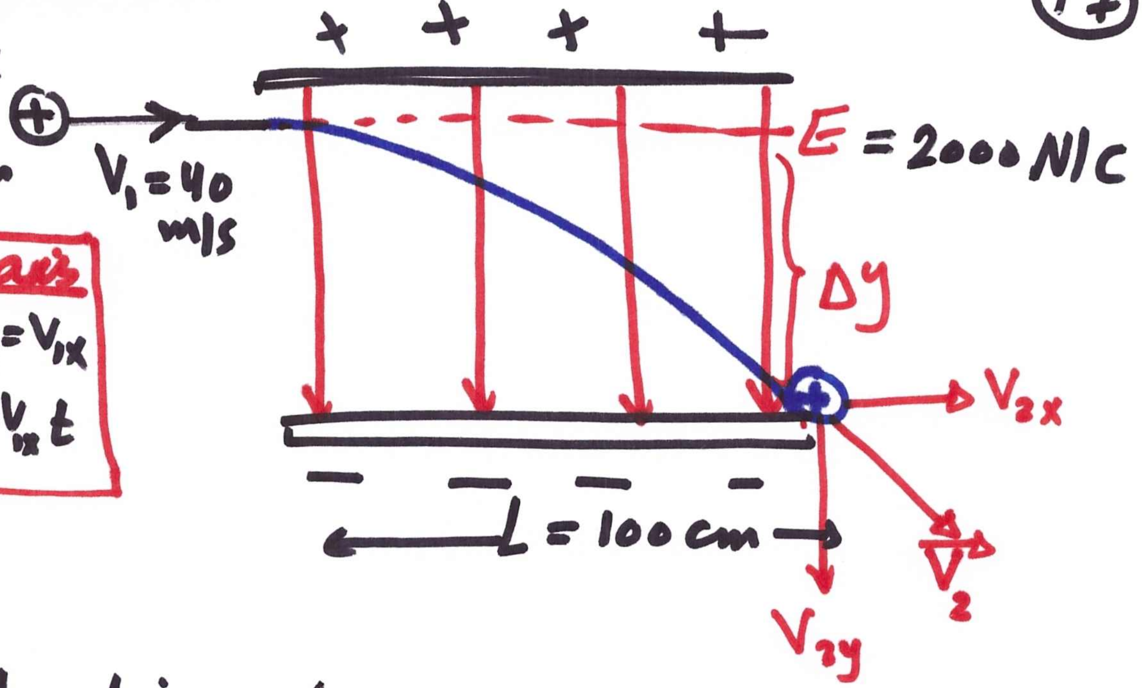
$$= 50\, N/C$$

* $V_2 = V_1 + at$

$$0 = 20 - 1t$$

$$\boxed{t = 20\ sec}$$

Ex: $q_0 = 10\,\mu C$
$m = 2\,gm$



y-axis	x-axis
$V_{2y} = V_{1y} + a_y t$	$V_{2x} = V_{1x}$
$V_{2y}^2 = V_{1y}^2 + 2a_y y$	$x = V_{1x} t$
$y = V_{1y} t + \frac{1}{2} a_y t^2$	

Find:

1) acceleration (a_y)
2) final velocity (speed).
3) Vertical displacement (Δy)
4) time.

$V_{1x} = V_1 \cos\theta = 40 \cos 0 = 40$ m/s

$V_{1y} = V_1 \sin\theta = 40 \sin 0 = 0$ m/s.

1) $q_0 E_y = m a_y$

$10 * 10^{-6} * 2000 = 2 * 10^{-3} a_y$

$\boxed{a_y = {}^{+}10\ m/s^2}$

2) $V_{2x} = V_{1x} = 40$

$V_{2y} = \cancel{V_{1y}} + a_y t$
$= 10 * 25 \times 10^{-3}$
$= 0.25$ m/s.

$x = V_{1x} t$
$1 = 40\, t$
$t = 25 \times 10^{-3}$ sec

$\vec{V}_2 = 40\hat{i} - 0.25\hat{j}$

$S = |\vec{V}_2| = \sqrt{40^2 + 0.25^2} =$

3) $\Delta y = \cancel{V_{1y}t} + \frac{1}{2} g_y t^2$

$= \frac{1}{2} * 10 * (0.025)^2$

$= 3.125 * 10^{-3}\ m$

$= 3.125\ mm.$

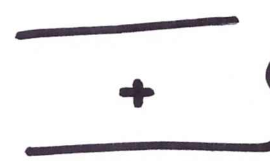
---

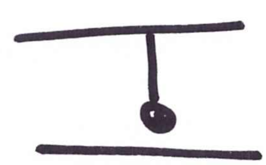
4) $t = 25 * 10^{-3}$ sec.

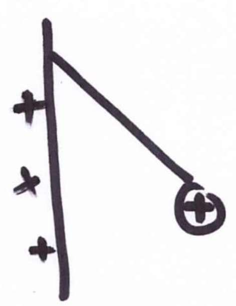
$\Delta y = V_{1y} t + \frac{1}{2} a t^2$

$\longrightarrow \Delta x = V_{1x} t$

$\boxed{٣}$ تطبيق اتزان الجسم المشحون داخل المجال المنتظم

١) 

٢) 

٣) 

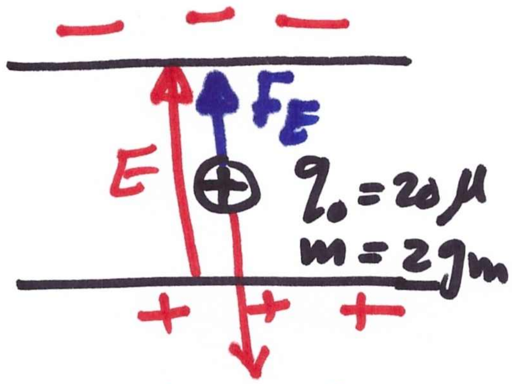
الخطوات :

١) نحدد اتجاه القوى المؤثرة .

٢) نحدد محاور مناسبة ونحلل القوة غير منطبقة على المحاور .

٣) نطبق قوانين الاتزان : $\sum F_{\uparrow} = \sum F_{\downarrow} , \sum F_{\rightarrow} = \sum F_{\leftarrow}$

Ex: particle of mass 2gm and charge of 20μc, what is the magnitude and direction of $\underline{E}$ when q_o is at equilibrium.



$w = mg$
$= 2 \times 10^{-3} \times 10$
$= 2 \times 10^{-2}$ N

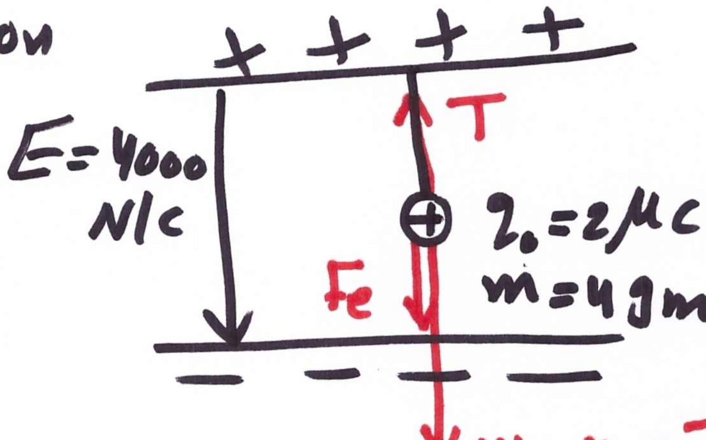
$$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$$

$$q_o E = w$$

$$20 \times 10^{-6} E = 2 \times 10^{-2}$$

$$\boxed{E = 1 \times 10^{3} \text{ N/C}}$$

Ex: what is the tension in the cord.



$w = 40 \times 10^{-3}$ N

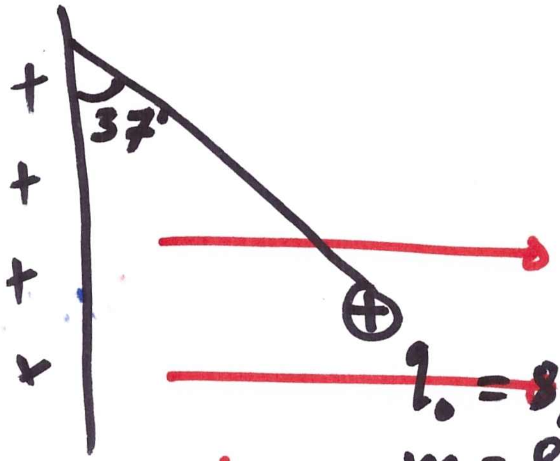
$$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$$

$$T = F_e + w$$

$$T = 48 \times 10^{-3} N.$$

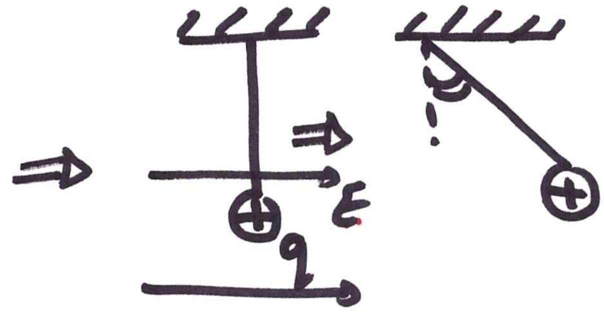
$F_e = q_o E$
$= 2 \times 10^{-6} \times 4 \times 10^{3}$
$= 8 \times 10^{-3}$ N

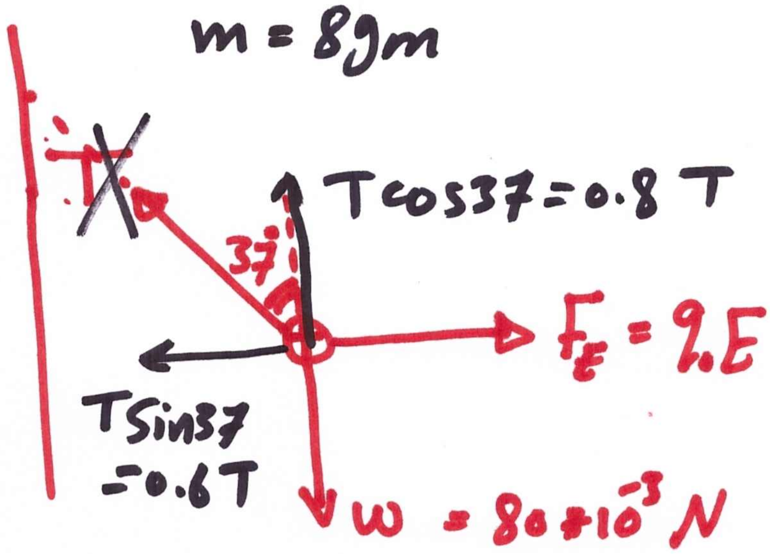
Ex : Find $\underline{E}$



$q_0 = 8\mu C$

$m = 8gm$





$$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$$

$$0.8\, T = 80 * 10^{-3}$$

$$\boxed{T = 0.1 \text{ New}}$$

$$\Sigma F_{\rightarrow} = \Sigma F_{\leftarrow}$$

$$q_0 E = T\, 0.6$$

$$E = \frac{0.6 * 0.1}{8 * 10^{-6}}$$

$$= \frac{6 * 10^{-2}}{8 * 10^{-6}} = 7.5 * 10^{3} \text{ N/c}$$

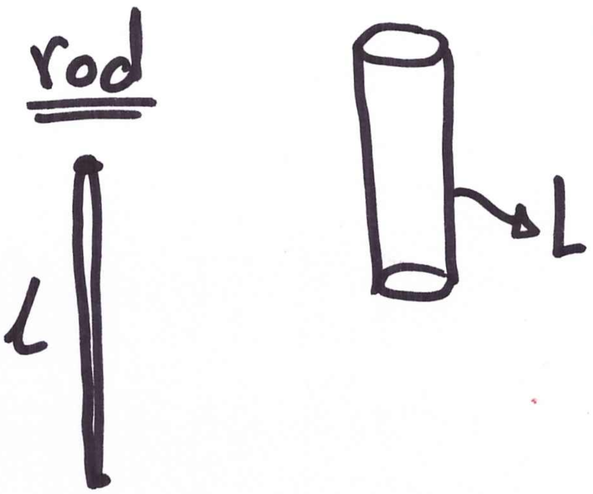
لو كانت معطاة

$E = 400 \text{ N/c} (-\hat{i})$

Electric Field due to distribution of charge.

المجال الناتج عن توزيع من الشحنات .

1) linear charge dist.

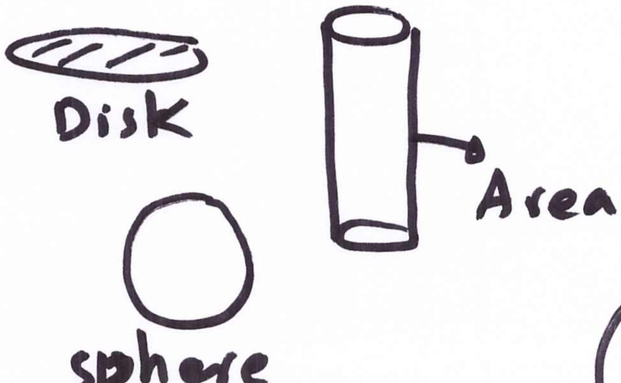


$\Rightarrow$ linear charge density (λ)
كثافة الشحنة الطولية.

$$\boxed{q = \lambda L}$$

$$dq = \lambda\, dL$$

2) Surface charge dist:-



$\Rightarrow$ surface charge density
كثافة الشحنة السطحية
(σ)

$$dq = \sigma\, dA \Longleftarrow \boxed{q = \sigma A}$$

3) Volume. charge dist.

$\Rightarrow$ Volume charge density (ρ) كثافة الشحنة الحجمية

$$dq = \rho\, dV \Longleftarrow q = \rho\, \text{Vol}$$

$D = 2r$

D → diameter

r → radius

$2\pi r$ = محيط الدائرة

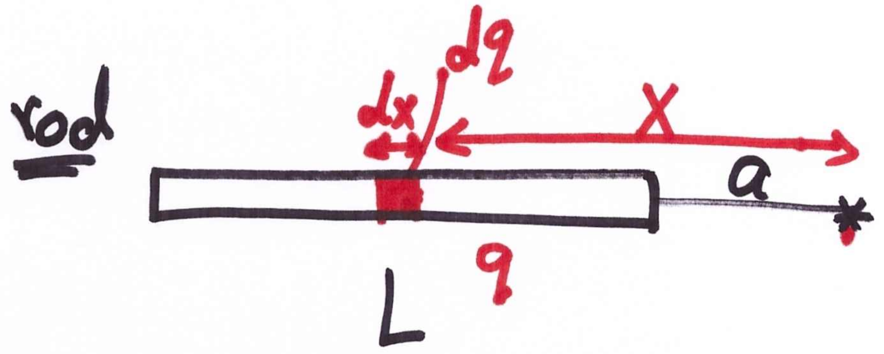
πr^2 = مساحة الدائرة

$4\pi r^2$ = مساحة الكرة

$\frac{4}{3}\pi r^3$ = حجم الكرة

$2\pi r l$ = المساحة الجانبية للاسطوانة

$\pi r^2 l$ = حجم الاسطوانة



$$q = \lambda L$$

$$dq = \lambda\, dx$$

$$\boxed{\lambda = \lambda_0 x}$$

$$dE = \frac{k\,dq}{r^2} = \frac{k\lambda\,dx}{x^2}$$

$$E = \int_{a}^{L+a} \frac{k\lambda\,dx}{x^2} = k\lambda \int_{a}^{l+a} \frac{dx}{x^2} = k\lambda \left[-\frac{1}{x}\right]_a^{L+a}$$

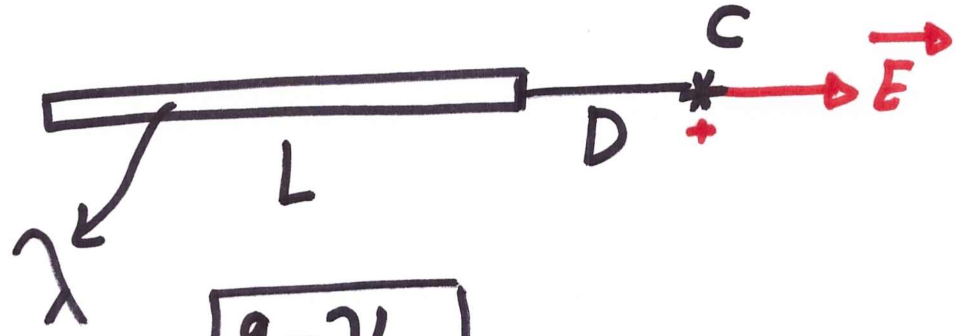
$$= k\lambda \left[\frac{-1}{L+a} - \frac{-1}{a}\right] = k\lambda\left[\frac{1}{a} - \frac{1}{L+a}\right]$$

$$= k\lambda\left[\frac{L+\cancel{a}-\cancel{a}}{a(L+a)}\right] = \frac{k\lambda L}{\ldots}$$

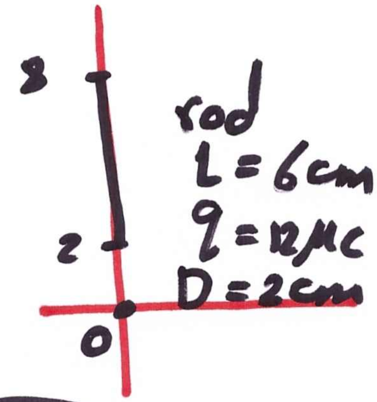
$\lambda L \rightarrow q$

1 rod:

a)

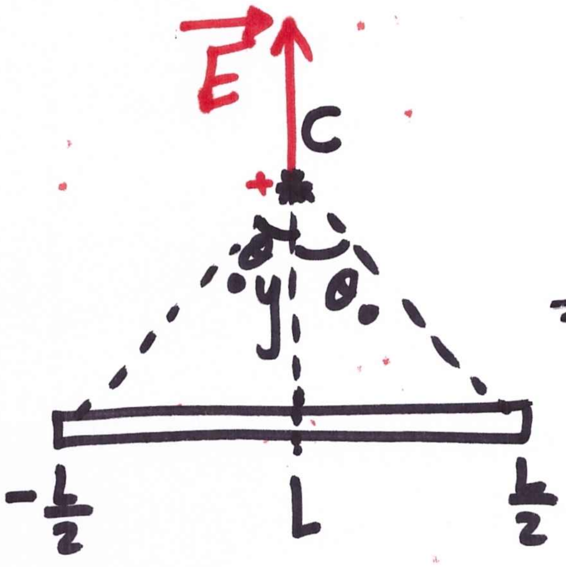


$$\boxed{q=\lambda L}$$



$$\Rightarrow E_c = \frac{k\lambda L}{D(D+L)} \qquad \lambda L \rightarrow q$$

b) finite rod

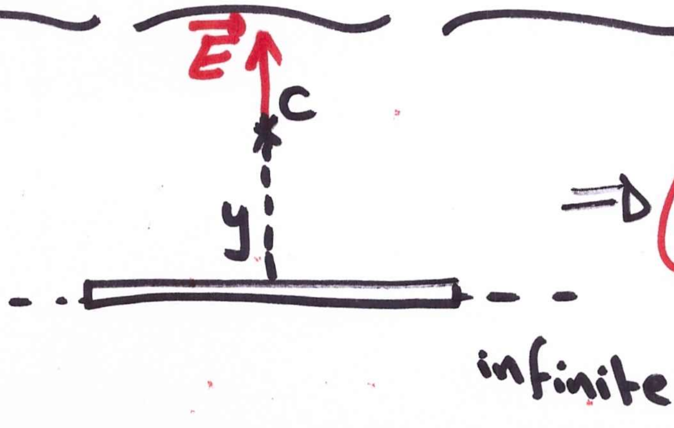


$$\Rightarrow E = \frac{2k\lambda \sin\theta_0}{y}$$

or

$$E = \frac{\cancel{2}k\lambda L}{\cancel{2}y\sqrt{y^2+\frac{L^2}{4}}}$$

c) infinite rod



$$\Rightarrow E_c = \frac{2k\lambda}{y}$$

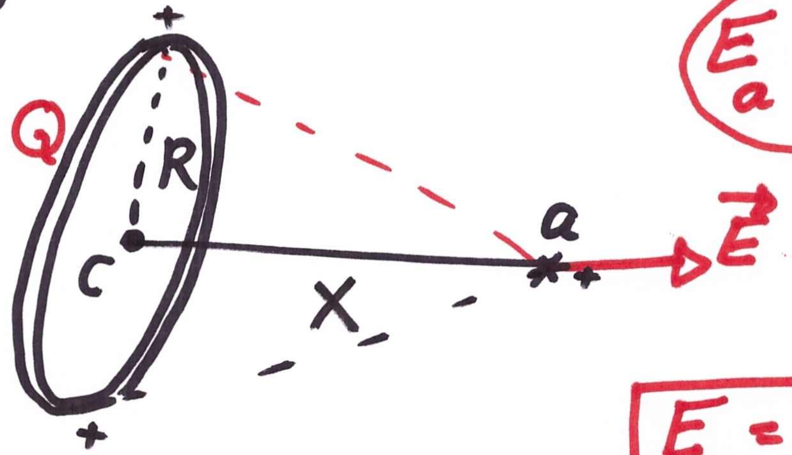
2 Ring

λ

$\downarrow$

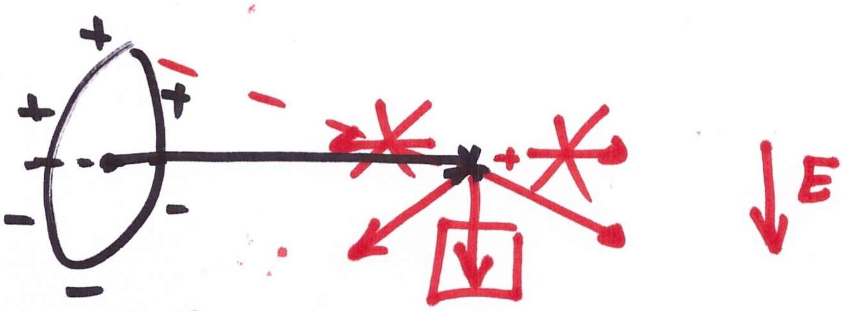
$Q = \lambda L$

$Q = \lambda (2\pi R)$

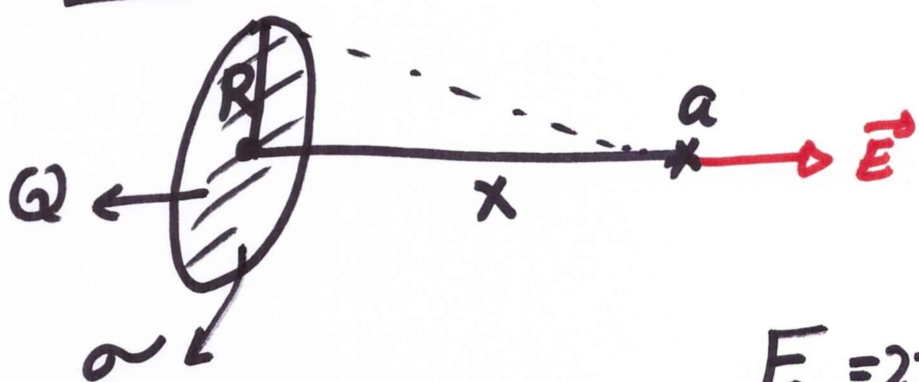


$$E_a = \frac{kQX}{(X^2+R^2)^{3/2}}$$

$$\boxed{E_c = 0}$$



3 Disk



$Q = \sigma A$

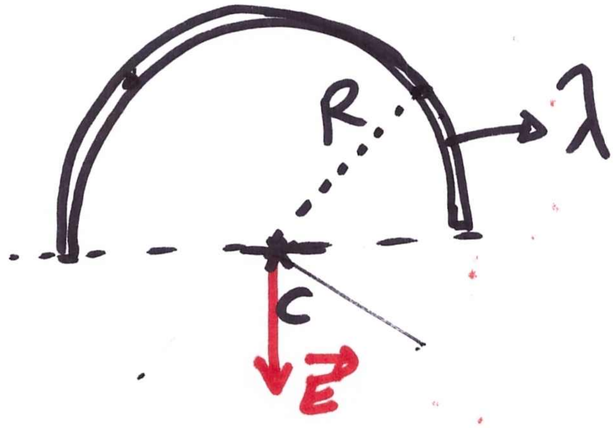
$= \sigma \pi R^2$

$$E = 2\pi k\sigma \left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$

$$E = 2\pi k\sigma \left[1 - \frac{X}{\sqrt{R^2+x^2}}\right]$$

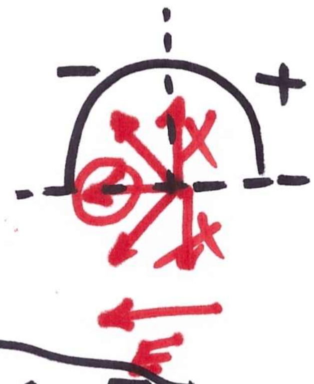
$$\sigma = \frac{Q}{\pi R^2}$$

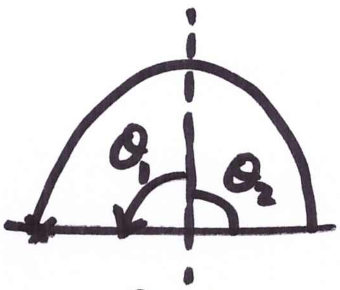
5) Semicircle :



$Q = \lambda L$

$$\boxed{Q = \lambda \pi R}$$

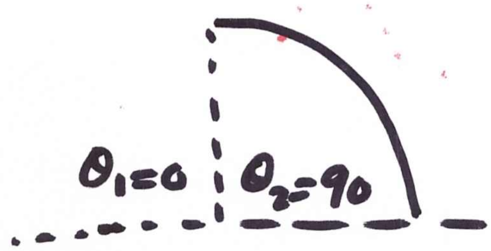




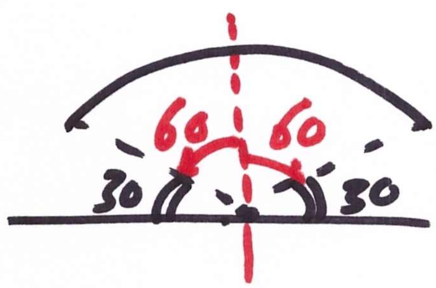
$\theta_1 = 90$

$\theta_2 = 90$

$$\boxed{E_c = \frac{k\lambda}{R}\left[\sin\theta_1 + \sin\theta_2\right]}$$



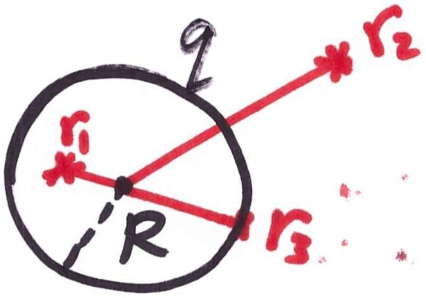
$\Rightarrow \frac{k\lambda}{R}[0+1]$



$\Rightarrow$ $\theta_1 = 60$

$\theta_2 = 60$

6 Conducting sphere Spherical shell.



$$q = \sigma (4\pi R^2)$$

- at $r_1 < R$

$$\boxed{E_{in} = 0}$$

- at $r_2 > R$

$$\boxed{E_{out} = \frac{kq}{r^2} = \frac{k\sigma 4\pi R^2}{r^2} = \frac{\sigma R^2}{\epsilon_0 r^2}}$$

- at $r_3 = R$

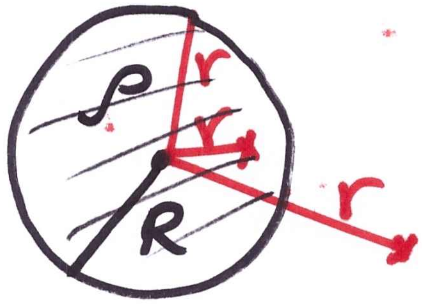
$$\boxed{E = \frac{kq}{R^2} = \frac{\sigma}{\epsilon_0}}$$

7 Solid insulator sphere

ليست فارغة   عازلة

① $r < R$ (in)

$$\boxed{E_{in} = \frac{kqr}{R^3} = \frac{\rho r}{3\epsilon_0}}$$



$$q = \rho\left(\frac{4}{3}\pi R^3\right)$$

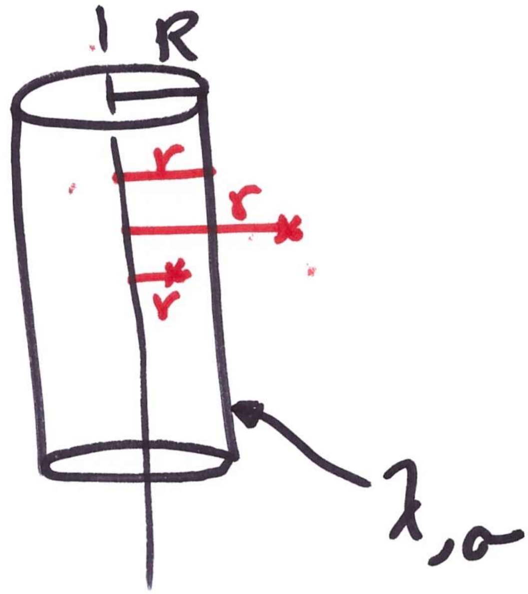
② $r > R$ (out)

$$E_{out} = \frac{kq}{r^2} = \frac{\rho R^3}{3\epsilon_0 r^2}$$

③ $r = R$

$$E = \frac{\rho R}{3\epsilon_0}$$

8 Conducting infinite cylinder cylindrical shell.



1) $r < R$ (in)

$$\boxed{E_{in} = 0}$$

2) $r > R$ (out)

$$E_{out} = \frac{\lambda}{2\pi \epsilon_0 r} = \frac{2k\lambda}{r}$$

$E(2\pi r) = \frac{\lambda L}{\epsilon}$

3) $r = R$

$$E = \frac{\lambda}{2\pi \epsilon_0 R}$$

$$Q = Q$$

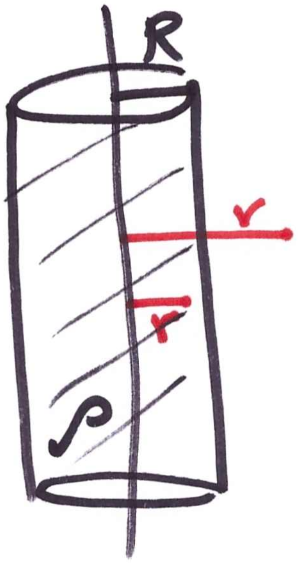
$$\sigma = \frac{\lambda}{2\pi R} \Leftarrow \lambda \cancel{L} = \sigma (2\pi R \cancel{L})$$

$$\Rightarrow E = \frac{\sigma}{\epsilon_0}, \quad E_{out} = \frac{R\sigma}{\epsilon_0 r}$$

9 Infinite Solid insulating cylinder

1) $r < R$ (in)

$$\boxed{E_{in} = \frac{\rho r}{2\epsilon_0}}$$



$\cancel{E(2\pi r l) = \frac{\rho \pi R^2 l}{2\epsilon_0}}$ $\cancel{E 2\pi r l = \frac{\rho 4 \pi r^2}{3\epsilon_0}}$

$E\, 2\pi r l = \frac{\rho \pi r^2 l}{\epsilon_0}$

$E = \frac{\rho r}{2\epsilon_0}$

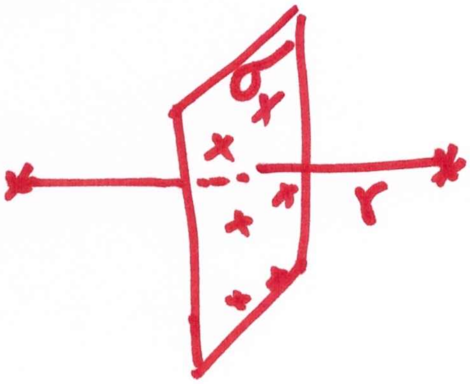
2) $r > R$

$$E_{out} = \frac{\rho R^2}{2\epsilon_0 r}$$

3) $r = R$

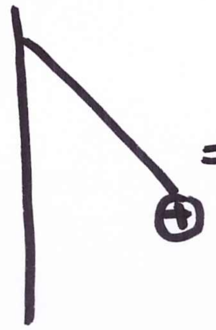
$$\boxed{E = \frac{\rho R}{2\epsilon_0}}$$

10 Infinite plate



$\Rightarrow E = \frac{\sigma}{2\epsilon_0}$ مهمة

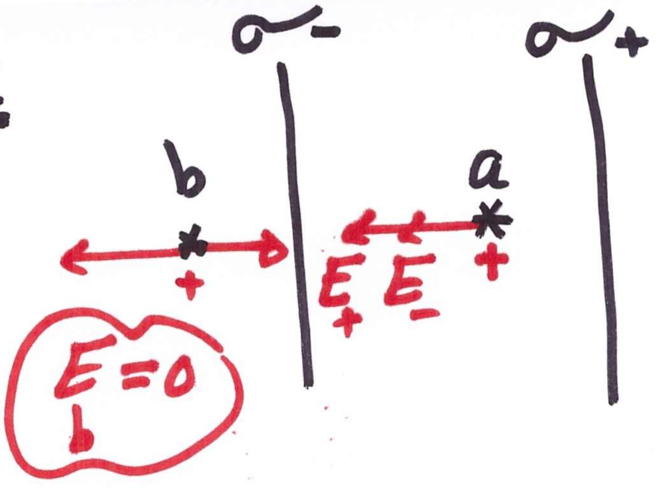
[1]



$\Rightarrow$ find E | find σ

$E = \checkmark$ | $E = \frac{\sigma}{2\epsilon_0} \Rightarrow \sigma = 2\epsilon_0 E$

2

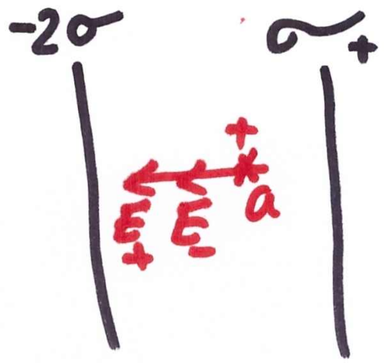


$$E_+ = \frac{\sigma}{2\epsilon_0}$$

$$E_- = \frac{\sigma}{2\epsilon_0}$$

$$\boxed{E_a = E_+ + E_- = \frac{\sigma}{\epsilon_0}}$$

3



$$E_+ = \frac{\sigma}{2\epsilon_0}$$

$$E_- = \frac{2\sigma}{2\epsilon_0}$$

$$E_a = \frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} = \frac{3\sigma}{2\epsilon_0}$$

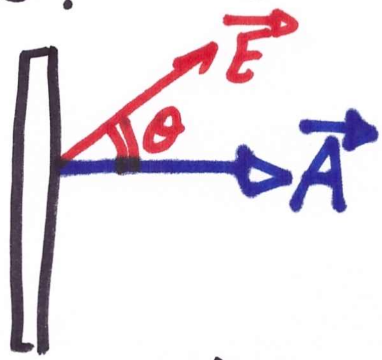
CH: 24 Electrical Flux and Gauss's law

Flux

Φ : عدد خطوط المجال التي تخترق سطحاً ما عمودياً عليه.

(Φ)

١) سطح منتظم يخترقه مجال منتظم



$$\Phi = \vec{E} \cdot \vec{A}$$
$$= EA\cos\theta_{EA}$$

٢) سطح مغلق بداخله شحنات

$$\Phi = \frac{\sum q_{in}}{\epsilon_0}$$

$$= \frac{q_1 + q_2}{\epsilon_0}$$

* السالب يعوض.

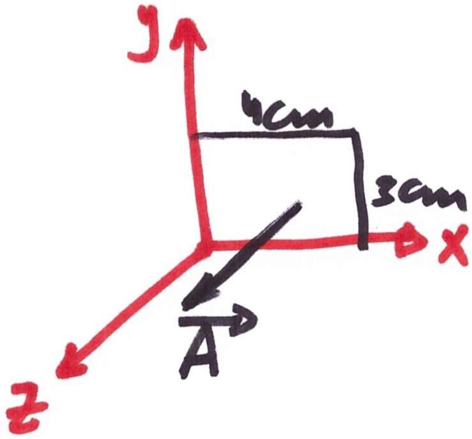
Ex: If $\vec{E} = 4\hat{i} + 2\hat{j} + 5\hat{k}$ and the Area is rectangular with dim. 4cm x 3cm lying on the xy ~~axis~~ plane. Find the electrical flux.

Sol:

$$A = 4 * 10^{-2} * 3 * 10^{-2}$$

$$A = 12 * 10^{-4} \ m^2$$

$$\vec{A} = 12 * 10^{-4} \ \hat{k}$$



$$\phi = \vec{E} \cdot \vec{A}$$

$$= (4\hat{i} + 2\hat{j} + 5\hat{k}) \cdot (12 \times 10^{-4} \ \hat{k})$$

$$= 60 * 10^{-4} \ N \cdot m^2/C.$$

Ex: If the $\underline{E}$ is given by: $\vec{E} = 4\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{A}$ is given by: $\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k}$, find

1) The net flux

$$\phi = \vec{E} \cdot \vec{A}$$

$$= 12 + 2 - 2$$

$$= 12 \ N \cdot m^2/C$$

2) angle between $\vec{E}$ and $\vec{A}$

$$\vec{E} \cdot \vec{A} = E A \cos\theta_{EA}$$

$$12 = \sqrt{16+1+4} \ \sqrt{9+4+1} \cos\theta$$

$$\cos\theta = \frac{12}{\sqrt{21} * \sqrt{14}} = 0.7$$

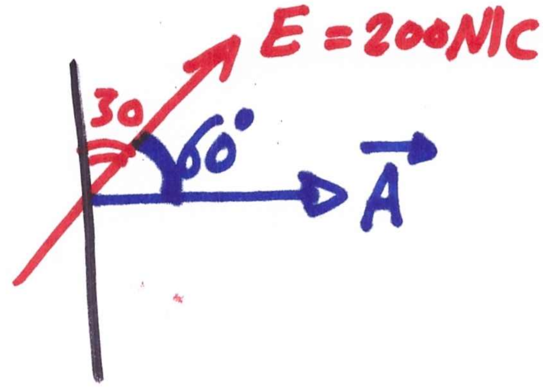
$$\boxed{\theta = 45.6^\circ}$$

Ex: In the figure, calculate the net flux

$\Phi = E A \cos\theta_{EA}$

$= 200 \times (2 \times 10^{-4}) \cos 60$

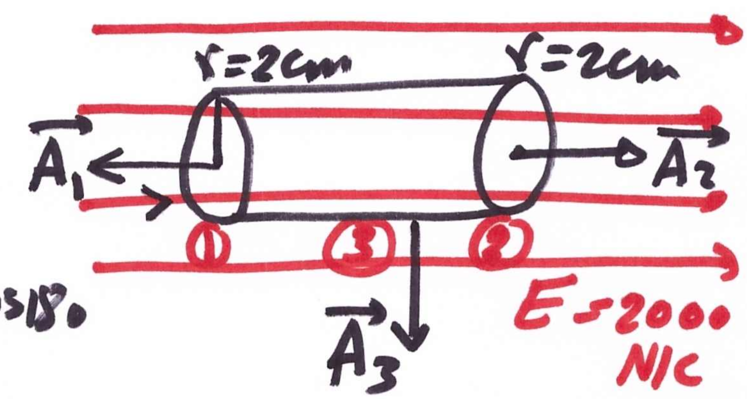
$= 0.02\ N.m^2/C.$



2cm x 1cm

Ex: Find the net Electrical Flux therough the below cylinder:

Sol



$\Phi_1 = E A \cos\theta$

$= 2 * 10^3 * \pi (2 \times 10^{-2})^2 \cos 180$

$= -8 \pi * 10^{-1}$

$= -0.8 \pi\ N.m^2/C$

$\Phi_2 = E A \cos\theta$

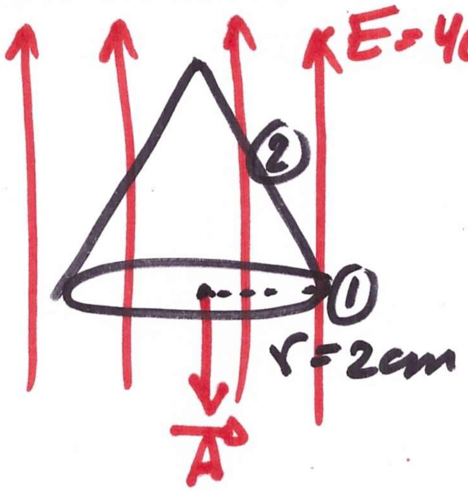
$= 2 \times 10^3\ \pi (2 * 10^{-2})^2 \cos 0$

$= +0.8 \pi\ N.m^2/C.$

$\Phi_3 = 2 * 10^3 * A_3 \cos 90 = 0$

$\Phi_{net} = -0.8\pi + 0.8\pi + 0 = 0$

Ex:



Find the Flux through each surface.

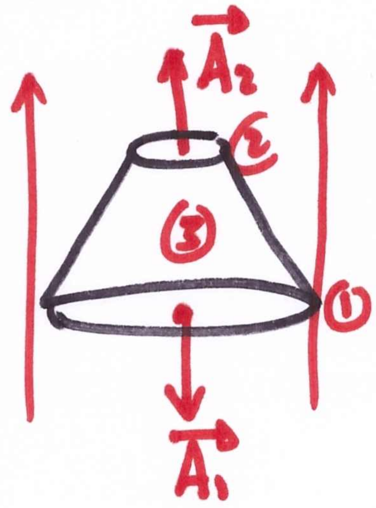
$$\Phi_1 = EA\cos\theta = 4\times10^{2}\times\pi(2\times10^{-2})^2\cos 180$$

$$= -16\pi\times10^{-2} = -0.16\,\pi\ N\cdot m^2/C$$

$$\Phi_{net} = \Phi_1 + \Phi_2$$

$$0 = -0.16 + \Phi_2 \Rightarrow \Phi_2 = +0.16\,N\,m^2/C.$$

Ex



$\Phi_1 = \dots\dots \cos 180 = -a$

$\Phi_2 = E A_2 \cos 0 = +b$

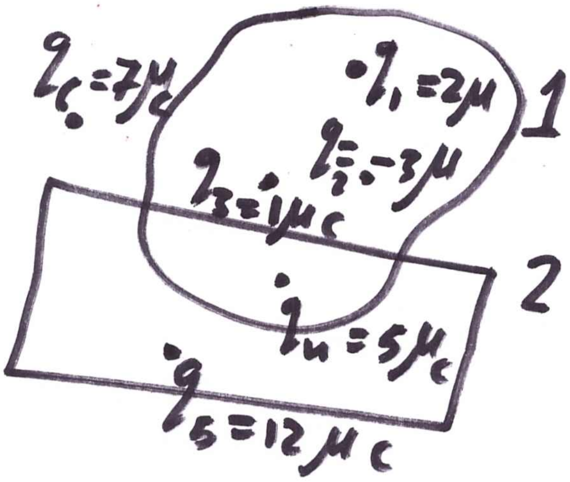
$\Phi_3 = ??$

$\Phi_{net} = \Phi_1 + \Phi_2 + \Phi_3$

$0 = -(a) + (b) + \Phi_3$

$\boxed{\Phi_3 = -b + a}$.

Ex: Find flux through each surface.



$$\Phi_1 = \frac{\sum q_{ins}}{\epsilon_0}$$

$$= \frac{2\mu + -3\mu + 1\mu + 5\mu}{\epsilon_0}$$

$$= \frac{5\times10^{-6}}{8.85\times10^{-12}} = 0.56\times10^{6}\ N\cdot m^2/C.$$

$$\Phi_2 = \frac{5\mu + 12\mu}{\epsilon_0} = \frac{17\times10^{-6}}{8.85\times10^{-12}} = 1.92\times10^{6}\ N\cdot m^2/C.$$

Ex: If we have two charges, $q_1 = 6\mu C$ at the origin, $q_2 = -4\mu C$ at $x = 2\,cm$.

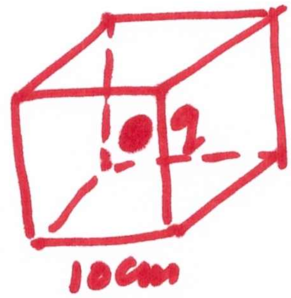
Find the net flux through a sphere of radius $r = 1\,cm$, centered at the origin.

$$\Phi = \frac{\sum q_{ins}}{\epsilon_0}$$

$$= \frac{6\times10^{-6}}{8.85\times10^{-12}} = 0.68\times10\ N\cdot m^2/C.$$

Ex: Cube of side 10 cm, Contains a charge at its center $q = 12\,\mu C$, Find:-

1) net flux.
2) Flux through each surface



1) $\Phi_{cube} = \dfrac{q}{\epsilon_0} = \dfrac{12\times10^{-6}}{8.85\times10^{-12}}$

$= 1.4\times10^{6}\ N.m^2/C.$

2) $\Phi_{face} = \dfrac{\Phi_{net}}{6} = 0.23\times10^{6}\ N.m^2/C.$

$$EA = \frac{\sum q_{ins}}{\epsilon_0} \Rightarrow \int E\,dA = \frac{q_{ins}}{\epsilon_0}$$

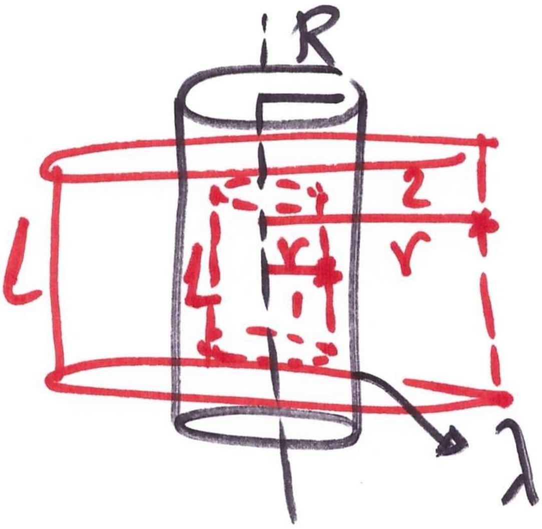
E: المجال المطلوب حسابه

A: مساحة سطح غاوس المحيط بالجسم

q_{ins}: الشحنة الكلية داخل سطح غاوس

$$q_{ins} = \begin{cases} \lambda L \rightarrow \int \lambda\,dx \\ \sigma A \rightarrow \int \sigma\,dA \\ \rho V \rightarrow \int \rho\,dv \\ q \\ 0 \end{cases}$$

Ex:



1) $EA = \frac{q_{ins}}{\epsilon_0}$

$E(2\pi r l) = \frac{0}{\epsilon_0}$

$E_{in} = 0$

2) $r > R$

$$EA = \frac{q_{ins}}{\epsilon_0} \Rightarrow E(2\pi r \cancel{l}) = \frac{\lambda \cancel{l}}{\epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

Ex

1) $r < R$

$$EA = \frac{q_{ins}}{\epsilon_0} \Rightarrow E(\cancel{4\pi} r^{\cancel{2}}) = \frac{\rho \frac{\cancel{4}}{3}\cancel{\pi} r^{3}}{\epsilon_0}$$

$$E = \frac{\rho r}{3\epsilon_0}$$

---

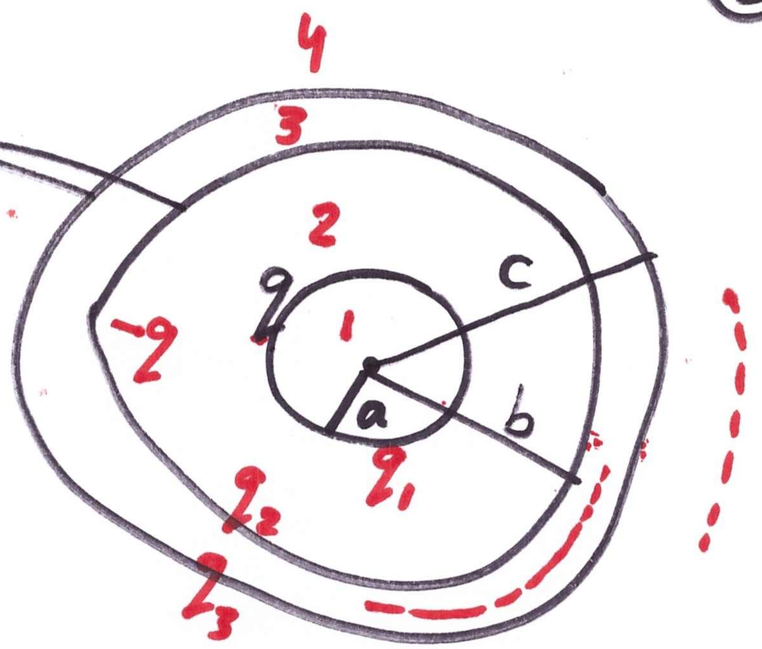
2) $r > R$

$$EA = \frac{q_{ins}}{\epsilon_0} \Rightarrow E(\cancel{4\pi} r^2) = \frac{\rho(\frac{\cancel{4}}{3}\cancel{\pi} R^3)}{\epsilon_0}$$

$$E = \frac{\rho R^3}{3\epsilon_0 r^2}.$$

Ex:

conducting thick shell



$$q_{shell} = \begin{Bmatrix} 0 \\ 2q \\ 5\mu C \end{Bmatrix}$$

$q_{shell} = q_2 + q_3$	$q_{shell} = q_2 + q_3$	$\boxed{q_2 = -q_1}$
$0 = -q + q_3$	$2q = -q + q_3$	
$\boxed{q_3 = q}$	$q_3 = 3q$	

Find $\underline{E}$ every where.

① $r < a$: $E_1 = \dfrac{\overset{\text{dile}}{\rho} r}{3\epsilon_0}$ | $E_1 = 0$

② $a < r < b$: $E_2 = \dfrac{kq}{r^2} \Rightarrow E_2 = \dfrac{\rho R^2}{3\epsilon_0 r^2}$

③ $c > r > b$: $EA = \dfrac{q_{ins}}{\epsilon_0}$

$$E(4\pi r^2) = \frac{(0)}{\epsilon_0}$$

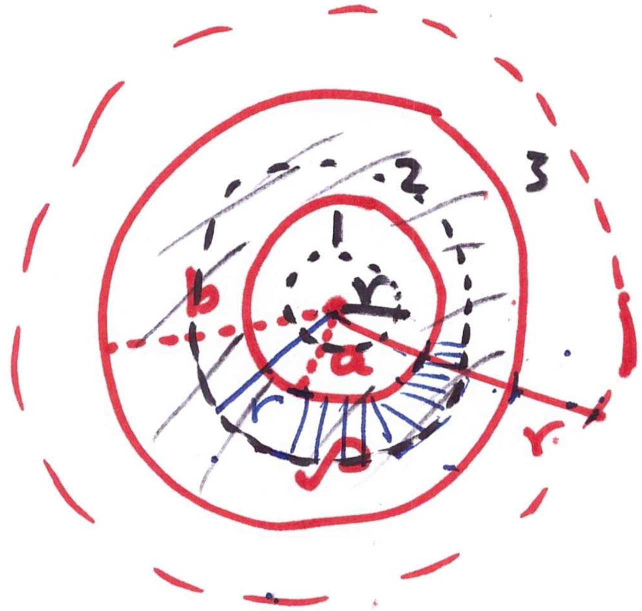
$$E_3 = 0$$

④ $r > c$

$$E(4\pi r^2) = \frac{q_3}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

Ex



1) $r < a$

$$EA = \frac{q_{ins}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{0}{\epsilon_0} \Rightarrow E_1 = 0$$

2) $a < r < b$

$$EA = \frac{q_{ins}}{\epsilon_0} \Rightarrow E(\cancel{4\pi} r^2) = \frac{\rho\left(\frac{\cancel{4}}{3}\cancel{\pi} r^3 - \frac{\cancel{4}}{3}\cancel{\pi} a^3\right)}{\epsilon_0}$$

$$E = \frac{\rho(r^3 - a^3)}{3\epsilon_0 r^2} = \frac{\rho\left(r - \frac{a^3}{r^2}\right)}{3\epsilon_0}$$

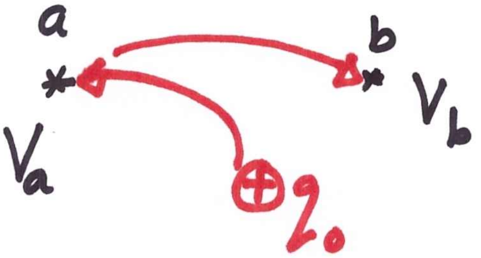
3) $r > b$

$$EA = \frac{q_{ins}}{\epsilon_0} \Rightarrow E(\cancel{4\pi} r^2) = \frac{\rho\left(\frac{\cancel{4}}{3}\cancel{\pi} b^3 - \frac{\cancel{4}}{3}\cancel{\pi} a^3\right)}{\epsilon_0}$$

$$\boxed{E_3 = \frac{\rho(b^3 - a^3)}{3\epsilon_0 r^2}}$$

CH:25 Electrical Potential

Volt ← (V) الجهد الكهربائي



Potential difference between a and b.
$\Rightarrow V_{ab} = V_a - V_b$

U : Potential energy

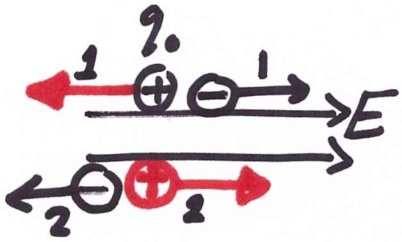
$$\left.\begin{aligned} U_a &= q_0 V_a \\ U_b &= q_0 V_b \end{aligned}\right] \Rightarrow \Delta U_{a\to b} = U_b - U_a = q_0 [V_b - V_a] = q_0 \Delta V_{a\to b} = q_0 V_{ba}$$

[1] V due to Point charge

$$W_{a\to b} = \Delta U_{a\to b} = q_0 \Delta V_{a\to b} = q_0 V_{ba} \quad [\Delta K = 0]$$

[2] V due to Uniform E field:

① $W_{a\to b} = \Delta U_{a\to b} = q_0 \Delta V_{a\to b} \quad [\Delta K = 0]$



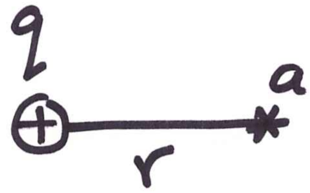
② $W_{a\to b} = \underset{\left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right)}{\Delta K} = -\Delta U_{a\to b} = -q_0 \Delta V_{a\to b}$

[3] V due to dist. of charges:

$$W_{a\to b} = \Delta U_{a\to b} = q_0 \Delta V_{a\to b} \quad [\Delta K = 0]$$

1 Electrical Potential due to Point charge

$$V = \frac{kq}{r}$$

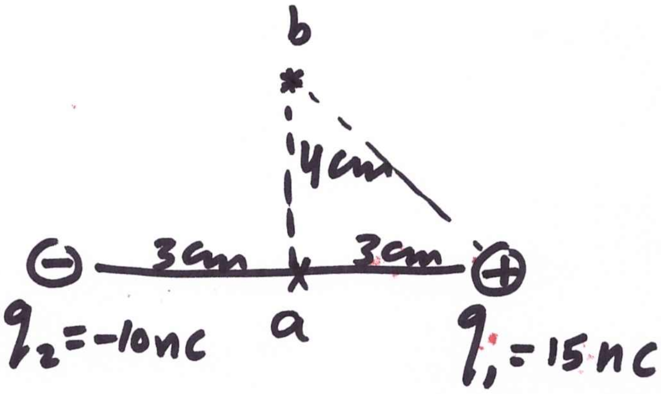


* السالب تعوضه .
* الجهد ليس له اتجاه .
* لحساب الجهد الكلي : $V_{total} = V_1 + V_2 + V_3 + \cdots$

Ex: In the figure, Find:

1) Potential difference between a and b.
2) Work needed to bring $q_0 = 2\mu C$ from a to b.
3) " " " " " " a to ∞

Sol



1) $V_a = V_1 + V_2$

$$= \frac{9\times10^{9} \times 15\times10^{-9}}{3\times10^{-2}} + \frac{9\times10^{9}\times -10\times10^{-9}}{3\times10^{-2}}$$

$$V_a = 45\times10^{2} - 30\times10^{2} = 15\times10^{2} \text{ Volt}.$$

$$V_b = \frac{9\times10^{9}\times 15\times10^{-9}}{5\times10^{-2}} + \frac{9\times10^{9}\times -10\times10^{-9}}{5\times10^{-2}} = (27-18)\times10^{2}$$
$$= 9\times10^{2} \text{ Volt}$$

$$V_{ab} = V_a - V_b$$

$$= 15\times10^2 - 9\times10^2 = 6*10^2 \text{ Volt.}$$

$$\Delta V_{a\to b} = V_{ba} = -V_{ab} = -6*10^2 \text{ Volt.}$$

---

2) $$W_{a\to b} = q_0 \Delta V_{a\to b}$$

$$= 2*10^{-6} * [-6*10^2]$$

$$= -12*10^{-4} \text{ J.}$$

---

3) $$W_{a\to\infty} = q_0 [V_\infty - V_a]$$

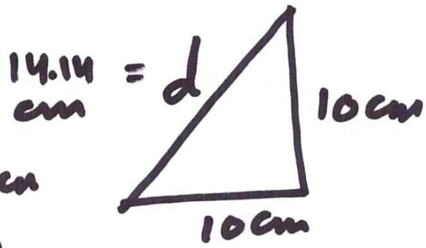
$$= 2*10^{-6} [0 - 15\times10^2]$$

$$= -30*10^{-4} \text{ J.}$$

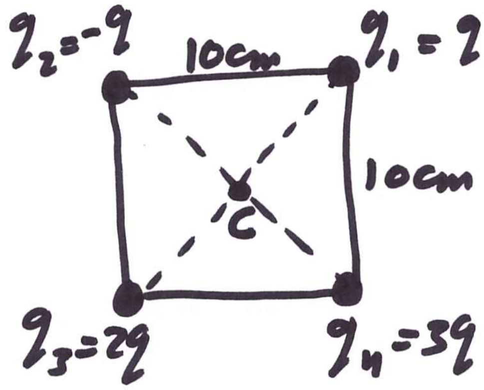
$\boxed{4}$ what is the potential at the Position of q_1.

$$V_{at\,q_1} = \frac{9*10^9 * -10*10^{-9}}{6*10^{-2}} = -15*10^2 \text{ Volt.}$$

Ex: Find the Electrical Potential at $\underline{C}$
$(q = 7\,nC)$



$\frac{1}{2}d = 7.07\,cm$
$\simeq 7\,cm$



$$V_c = V_1 + V_2 + V_3 + V_4$$

$$= \cancel{\frac{9*10^9 * 7*10^{-9}}{7*10^{-2}}} + \cancel{\frac{9*10^9 * -7\times10^{-9}}{7\times10^{-2}}} + \frac{9\times10^9\times14*10^{-9}}{7\times10^{-2}}$$

$$+ \frac{9\times10^9 * 21*10^{-9}}{7\times10^{-2}}$$

$$V_c = \frac{9\times10^9\times10^{-9}}{7\times10^{-2}}\left[14 + 21\right] = 45*10^2 \text{ Volt}$$

b) If we want the Work to bring an $\underline{\text{electron}}$ from ∞ to C:

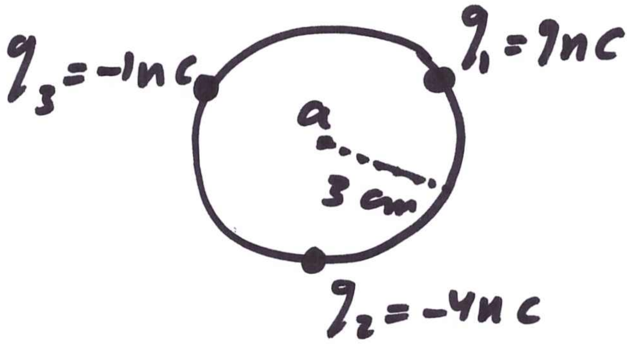
$q_e = -1.6*10^{-19}\,C$

$$W = q_0\left[V_c - \cancel{V_\infty}\right]$$

$$= -1.6\times10^{-19}\left[45\times10^2 - 0\right]$$

$$= -72*10^{-17}\,J.$$

Ex: what is the Work needed to bring a proton from ∞ to $\underline{\underline{a}}$.



$$\Rightarrow W = q_0 [V_a - \cancel{V_\infty}]$$

$$= +1.6 \times 10^{-19} [V_a]$$

$$= 1.6 \times 10^{-19} \times 12 \times 10^{-2} = 19.2 \times 10^{-21} \text{ J}.$$

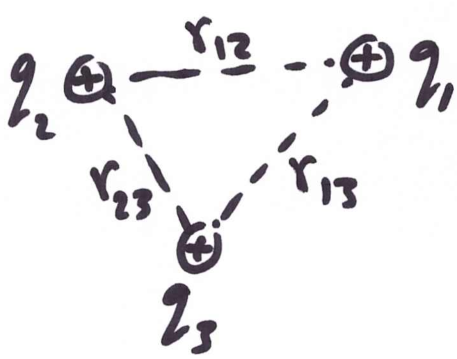
$$V_a = V_1 + V_2 + V_3$$

$$= \frac{9 \times 10^9}{3 \times 10^{-2}} [9 \times 10^{-9} + -4 \times 10^{-9} + -1 \times 10^{-9}]$$

$$= 3 \times 10^7 [4 \times 10^{-9}] = 12 \times 10^{-2} \text{ Volt}.$$

---

* Energy stored in $\underline{\underline{q_1}}$ فقط :



$$U_1 = U_{12} + U_{13}$$

$$= \frac{k q_1 q_2}{r_{12}} + \frac{k q_1 q_3}{r_{13}}$$

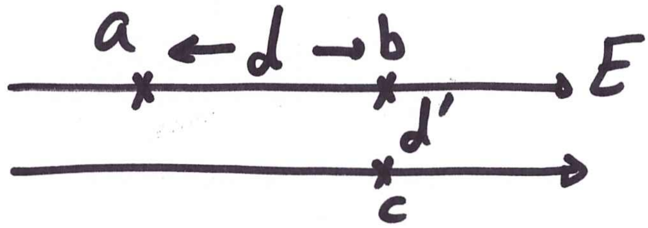
* Energy stored in the system :

$$U_{sys} = U_{12} + U_{13} + U_{23} \longrightarrow \frac{k q_2 q_3}{r_{23}}$$

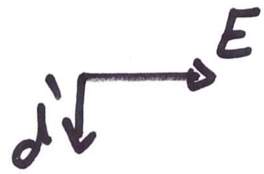
2 Electric Potential due Uniform $\underline{E}$:

$$V_{ba} = \Delta V_{a\to b} = -Ed\underset{Ed}{\cos\theta}$$

$$= -\vec{E}\cdot\vec{d}$$



* $\Delta V_{b\to c} = -E(d')\cos 90$



$\Delta V_{b\to c} = 0$

$V_c - V_b = 0 \Rightarrow \boxed{V_c = V_b}$

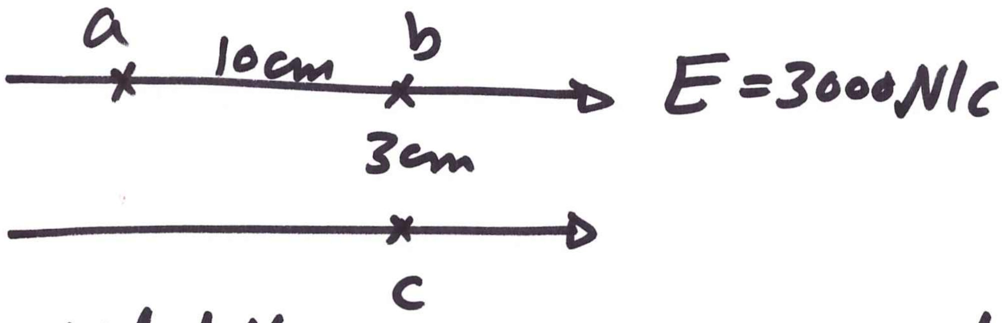
* $\Delta V_{a\to c} = \Delta V_{a\to b} + \cancel{\Delta V_{b\to c}}$

* if we know $V_a \Rightarrow$ what is V_b:

$\Delta V_{a\to b} = \checkmark \Rightarrow V_b - V_a = \checkmark$

$V_b = V_a + \checkmark$.

Ex:



1) Find Potential difference between a and b.
b " c
a " c.

2) Find the Work needed to move $q_0 = 2\mu C$ from a to b.

3) if q_0 in Part(2) started from rest ($m = 2gm$), what is its final speed.

4) Find the Work needed to move $q_0 = 5\mu c$ from b to a.

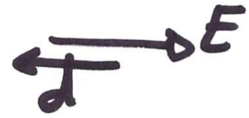
5) Find the Work needed to move $q_0 = -1nc$ from a to b.

6) if $V_a = 20$ Volt, what is V_b.

7*) What is the Work needed to move q_0 from $a \to a$ along the path $a \to b \to c \to a$

Sol :

1) $V_{ab} = \Delta V_{b\to a} = -Ed\cos\theta_{a\to b}$



$= -3\times10^{3} * 10*10^{-2} * \cos 180$

$= 300$ Volt .

* $\Delta V_{a\to b} = -300$ Volt.

2) * $\Delta V_{c\to b} = 0$

* $\Delta V_{ac} \Rightarrow \Delta V_{c\to a} = \cancel{\Delta V_{c\to b}} + \Delta V_{b\to a}$

$= 300$ Volt.

2) $W = -q_o \Delta V_{a\to b}$

$= -2*10^{-6}[-300] = 600*10^{-6}$ J

3) $\Delta K_{a\to b} = -q_o \Delta V_{a\to b}$

$\frac{1}{2}mV_b^2 - \cancel{\frac{1}{2}mV_a^2} = 600*10^{-6}$

$\frac{1}{\cancel{2}} * \cancel{2}*10^{-3} * V_b^2 = 600*10^{-6} \Rightarrow V_b = 0.77$ m/s.

(4) $W_{b\to a} = q_0 \, \Delta V_{b\to a}$

$= 5 * 10^{-6} [300]$

$= 1500 * 10^{-6}$ J.

(5) $W_{a\to b} = q_0 \, \Delta V_{a\to b}$

$= -1 * 10^{-9} * -300$

$= 300 * 10^{-9}$ J.

(6) $\Delta V_{a\to b} = -300$

$V_b - V_a = -300$

$V_b - 20 = -300 \Rightarrow V_b = -280$ Volt.

(7) $W_{a\to a} = q_0 \, \Delta V_{a\to a}$

$= q_0 [\Delta V_{a\to b} + \Delta V_{b\to c} + \Delta V_{c\to a}]$

$= q_0 [-300 + 0 + 300]$

$=$ Zero.

3 Potential due to distribution of charges (1o

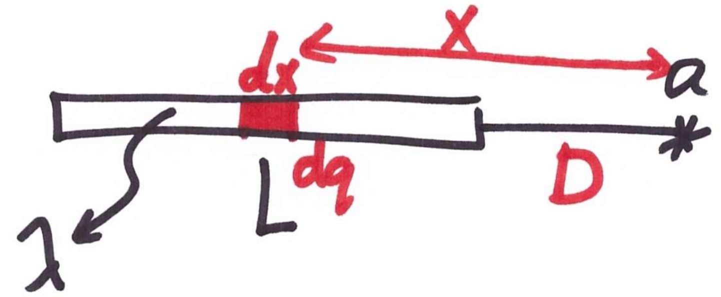
الجهد الناتج عن توزيع من الشحنات

$q = \lambda L \quad , \quad q = \sigma A \quad , \quad q = \rho V$

$dq = \lambda dx$

<u>Ex</u>:

$q = \lambda L$



$$V = \frac{kq}{r} \Rightarrow dV = \int \frac{k\,dq}{r} = \int \frac{k\lambda\,dx}{r = x}$$

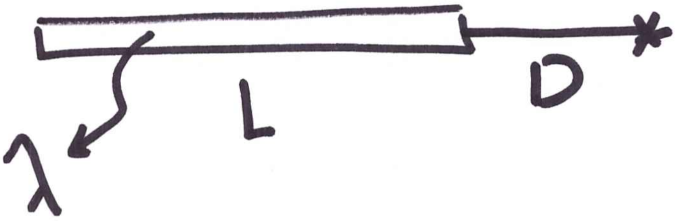
$$dV = k\lambda \int_{D}^{D+L} \frac{dx}{x} = k\lambda \ln x \Big|_{D}^{D+L}$$

$$= k\lambda \left[\ln(D+L) - \ln D\right]$$

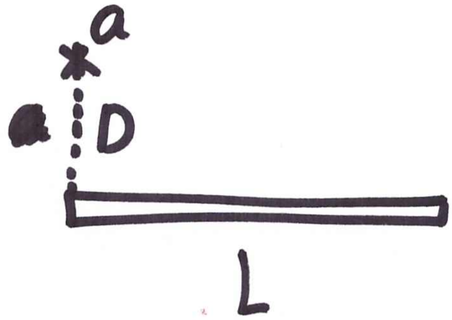
$$= k\lambda \ln\left(\frac{D+L}{D}\right)$$

1 rod:

$q = \lambda L$



$$V_a = k\lambda \ln\left(\frac{D+L}{D}\right)$$

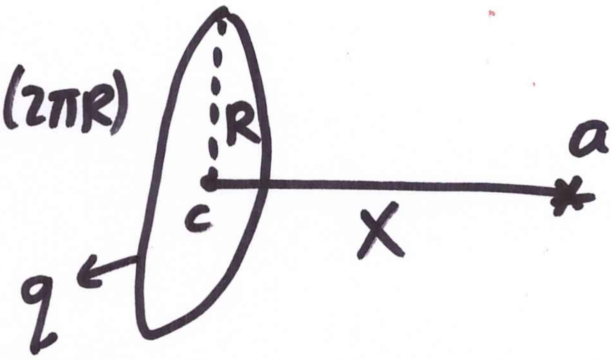


$$\Rightarrow V_a = k\lambda \ln\left[\frac{L+\sqrt{D^2+L^2}}{D}\right]$$

---

2 ring:

$q = \lambda(2\pi R)$

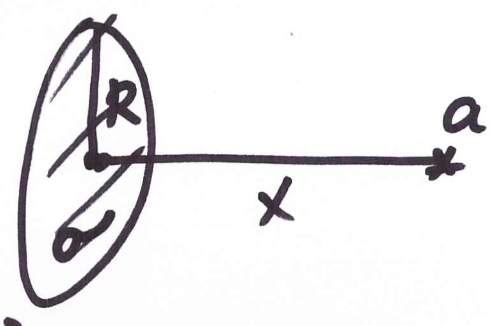


$$V_a = \frac{kq}{\sqrt{R^2+x^2}}$$

$$V_c = \frac{kq}{R}$$

---

3 Disk

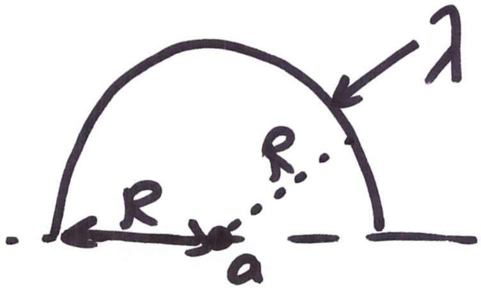


$$V_a = 2\pi \underset{9\times10^9}{k} \sigma\left[\sqrt{R^2+x^2} - x\right]$$

$q = \sigma \pi R^2$

4 Semicircle:

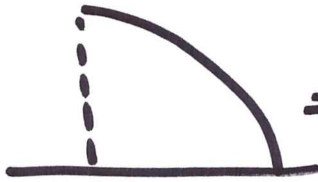
$q = \lambda \pi R$



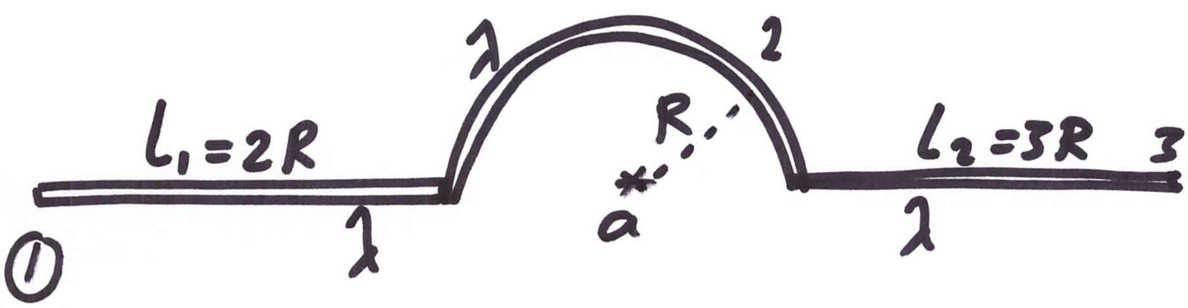
$V_a = k \lambda \theta$

$\downarrow$

$3.14 = \pi$



$\Rightarrow \theta = \frac{\pi}{2} \Rightarrow V = \frac{k \lambda \pi}{2}$



$$\Rightarrow V_a = V_1 + V_2 + V_3 .$$

$$= k\lambda \ln\left(\frac{D+l}{D}\right) + k\lambda\theta + k\lambda \ln\left(\frac{D+l}{D}\right)$$

$$= k\lambda \left(\ln\left(\frac{R+2R}{R}\right) + k\lambda\pi + k\lambda \ln\left(\frac{R+3R}{R}\right)\right.$$

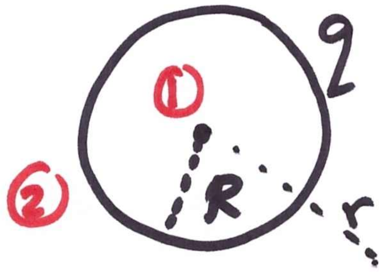
$$= k\lambda \ln 3 + k\lambda\pi + k\lambda \ln 4$$

$$= (\quad \bar{r}, \quad)$$

5 Conducting sphere:-

1) $r < R$ (inside)

$$V_{in} = \frac{kq}{R} = V_{surface}$$

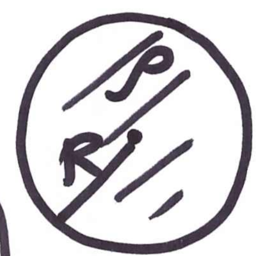


$q = \sigma \, 4\pi R^2$

2) $r > R$ (outside)

$$V_{out} = \frac{kq}{r}$$

6 insulating sphere



$q = \rho \frac{4}{3}\pi R^3$

1) $r < R \Rightarrow V_{in} = \frac{kq}{2R}\left(3 - \frac{r^2}{R^2}\right)$

2) $r \geq R \Rightarrow V_{out} = \frac{kq}{r}$

$$\Delta U = W = q_0 \Delta V$$

اذا اعطاك في السؤال $\vec{E}$ وطلب V :

(معادلة)

$$\boxed{\Delta V = -\int E_i\, dx - \int E_j\, dy - \int E_k\, dz - \int E\, dr}$$

Ex: If $E = \frac{4}{r^2}$, Find $\underline{V}$ from ∞ to 5cm

sol

$$\Rightarrow V = -\int E\, dr$$

$$= -\int_{\infty}^{5\,cm} \frac{4}{r^2}\, dr = -4\left[\frac{-1}{r}\right]_{\infty}^{5\,cm}$$

$$= \frac{4}{5\times 10^{-2}} - 0 = 80 \text{ Volt.}$$

Ex: If $\vec{E} = 3x^2yz\,\hat{i} - 5xz\,\hat{j} + 2zy\,\hat{k}$

Find $\underline{V}$

$$V = -\int 3x^2yz\, dx - \int -5xz\, dy - \int 2zy\, dz$$

$$= -3yz\frac{x^3}{3} + 5xzy - 2y\frac{z^2}{2}$$

$$= -yzx^3 + 5xyz - yz^2 .$$

$(1, 1, -2)$

إذا أعطاك $\underline{V}$ على شكل معادلة وطلب $\vec{E}$:

$$\vec{E} = -\hat{i}\,\frac{\partial V}{\partial x} - \hat{j}\,\frac{\partial V}{\partial y} - \hat{k}\,\frac{\partial V}{\partial z} - \hat{r}\,\frac{\partial V}{\partial r}$$

Ex : If $V = 2x^2yz^3 + 5xz$, Find:-

1) $\vec{E}$ at $(1,2,-1)$
2) magnitude of $\vec{E}$ at $(1,2,-1)$
3) if $q_0 = 2\mu C$, Find Force.

Sol :

1) $\dfrac{\partial V}{\partial x} = 4xyz^3 + 5z$ | $\dfrac{\partial V}{\partial y} = 2x^2z^3$ | $\dfrac{\partial V}{\partial z} = 6x^2yz^2 + 5x$

$\Rightarrow \dfrac{\partial V}{\partial x} = -8 + -5 = -13$

$\dfrac{\partial V}{\partial y} = -2$ | $\dfrac{\partial V}{\partial z} = 12 + 5 = 17$

$\Rightarrow \vec{E} = -(-13)\hat{i} - (-2)\hat{j} - (17)\hat{k}$

$$\boxed{\vec{E} = 13\hat{i} + 2\hat{j} - 17\hat{k}}$$

2) $|\vec{E}| = \sqrt{13^2 + 2^2 + 17^2}$ N/C

3) $\vec{F} = q_0\vec{E} = 2 \times 10^{-6}(13\hat{i} + 2\hat{j} - 17\hat{k})$.

انتهى CH:25

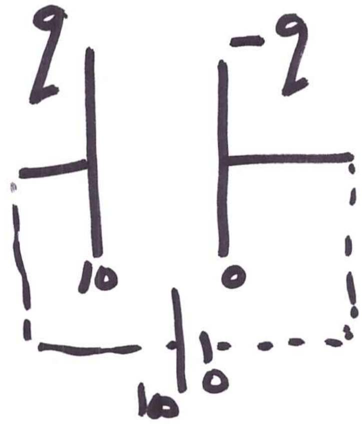
CH:26 Capacitors

$C \equiv$ Capacitance
المواسعة الكهربائية

$$\boxed{C = \frac{q}{\Delta V}}$$

عام

$$\Delta V = V_+ - V_-$$

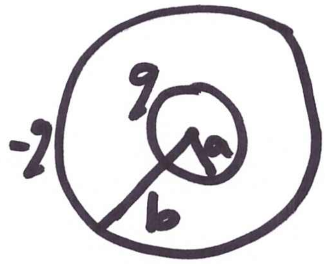


* أي جسمين متلامسين يصبح لهما نفس الجهد.

Farad

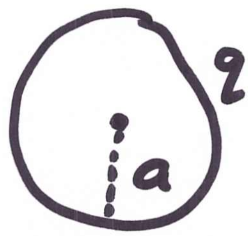
① Spherical capacitor:

$$\boxed{C = \frac{4\pi\epsilon_0}{\frac{1}{a} - \frac{1}{b}}}$$

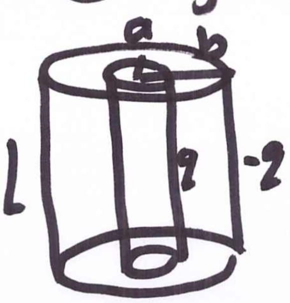


② sphere:

$$\boxed{C = 4\pi\epsilon_0 a}$$



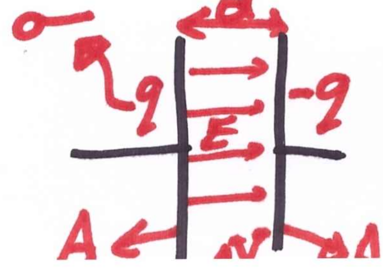
③ cylindrical capacitor



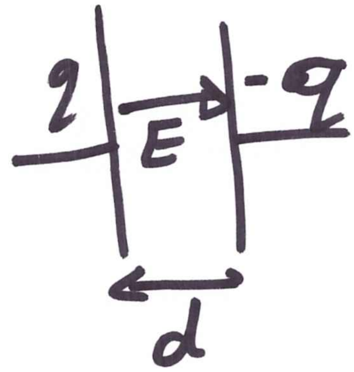
$$C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$

④ Two Parallel Plates Capa
مكثف ذو لوحين متوازيين

$$C = \frac{\epsilon_0 A}{d}$$



$C = \frac{\epsilon_0 A}{d}$ $\quad E = \frac{\sigma}{\epsilon_0}$



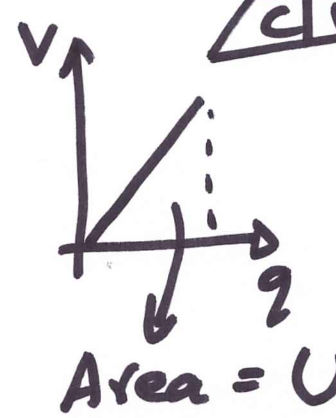
$q = \sigma A$

$C = \frac{q}{\Delta V}$ $\quad \Delta V = E d$

لوحين لوحين

* Energy stored in capacitor :

$$\boxed{U = \frac{1}{2} q V = \frac{1}{2} C V^2 = \frac{1}{2} \frac{q^2}{C}}$$



$W = \Delta U = U_f - U_i$

$u \equiv$ Energy density (كثافة طاقة)

energy per Unit Volume.

الطاقة في وحدة الحجوم.

$$\boxed{u = \frac{U}{\text{Volume}} = \frac{1}{2} \epsilon_0 E^2}$$

Volume → Ad

Ex: Two Parallel Plates Capacitor has a surface charge density $(\sigma) = 2*10^{-6}\ C/m^2$ and Area of 2 cm² with separation 10 cm,

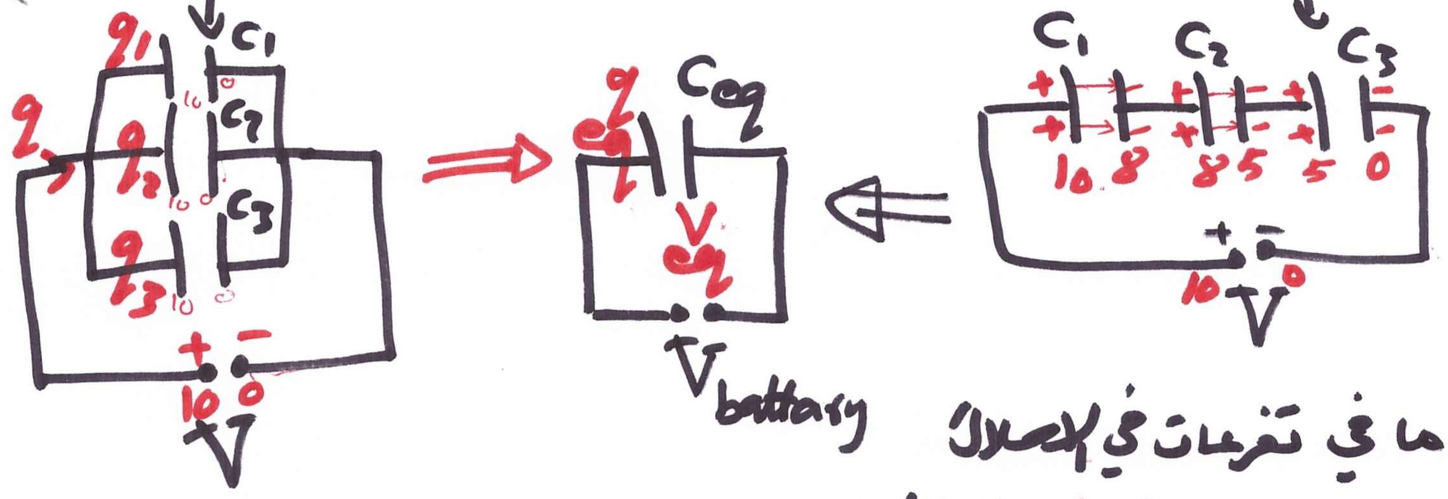
$A = 2*10^{-4}\ m^2$ البعد بين اللوحين $d = 10\times10^{-2}\ m$

Find:-

1. Capacitance (c). $\Rightarrow C = \frac{\epsilon_0 A}{d}$
2. charge (q). $\Rightarrow q = \sigma A$
3. Potential difference accross the Cap. $\Rightarrow V = \frac{q}{c}$
4. Electric field. $\Rightarrow E = \frac{\sigma}{\epsilon_0}$ or $E = \frac{V}{d}$
5. Energy stored in Cap. $\Rightarrow U = \frac{1}{2} q V = \frac{1}{2} C V^2$
6. energy density. (u) $\Rightarrow u = \frac{1}{2}\epsilon_0 E^2 = \frac{U}{Ad}$

Connection of Capacitors

(المتوازي) in Parallel — (توالي) in Series



In Parallel

* يوجد تفرع في الأسلاك، لكن لكل تفرع مكثف واحد فقط والفروع لها نفس البداية والنهاية

* اللوح الموجب مع الموجب والسالب مع السالب

* تتوزع الشحنة الكهربائية:

$$q_{eq} = q_1 + q_2 + q_3 + \dots$$

* الجهد الكهربائي متساوي:

$$V_{bat} = V_{eq} = V_1 = V_2 = V_3 = \dots$$

* السعة مجموعة:

$$C_{eq} = C_1 + C_2 + C_3 + \dots$$

In Series

- ما في تفرعات في الأسلاك

* اللوح الموجب متصل مع اللوح السالب

* تكون الشحنة متساوية

$$q_{eq} = q_1 = q_2 = q_3 = \dots$$

* يتوزع الجهد الكهربائي

$$V_{eq} = V_{bat} = V_1 + V_2 + V_3 + \dots$$

* السعة مقلوبة

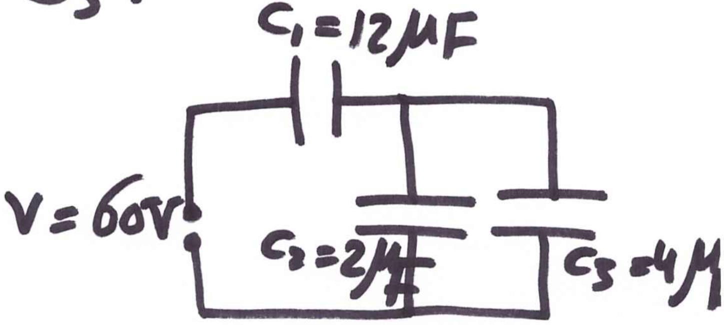
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

Ex: In the figure shown, Find:

① equivalent capacitance

② q, V for all capacitors.

③ energy stored in C_3.



Sol:

$C_2, C_3 \xrightarrow{\text{توازي}} C_4$

$C_1, C_4 \xrightarrow{\text{توالي}} C_5$

$V_5 = 60V$

* ارسم خريطة للسؤال ونضيف عليها المعلومة الإضافية
* نجد C_{eq}
* نجد جميع معلومات العنصر الإضافي
* من الخريطة نبدأ من العنصر الإضافي للوصول للمطلوب .

① $C_4 = C_2 + C_3$
$= 2\mu + 4\mu$
$= 6\mu F$

$\frac{1}{C_5} = \frac{1}{C_1} + \frac{1}{C_4}$

$\frac{1}{C_5} = \frac{1}{12\mu} + \frac{1}{6\mu}$

$\boxed{C_5 = 4\mu F}$

$\Downarrow$

$q_5 = C_5 V_5$

$\boxed{q_5 = 240\mu C}$

② $q_1 = q_4 = q_5 = 240\mu C$

$\rightarrow V_4 = \frac{q_4}{C_4} = 40\ \text{Volt}$

$\rightarrow V_1 = \frac{q_1}{C_1} = 20\ \text{Volt}.$

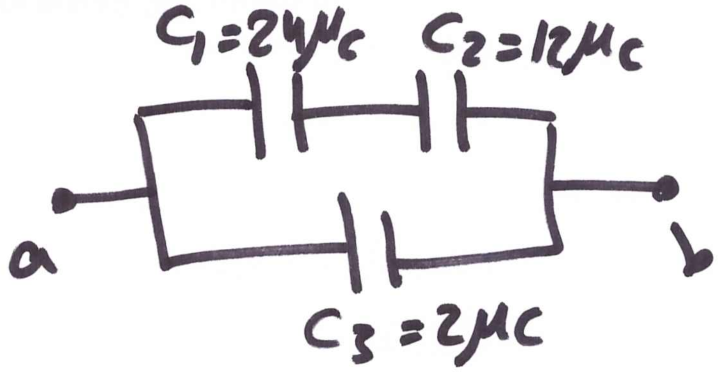
$V_2 = V_3 = V_4 = 40\ \text{Volt}$

$\rightarrow q_3 = C_3 V_3 = 160\mu C$

$\rightarrow q_2 = C_2 V_2 = 80\mu C$

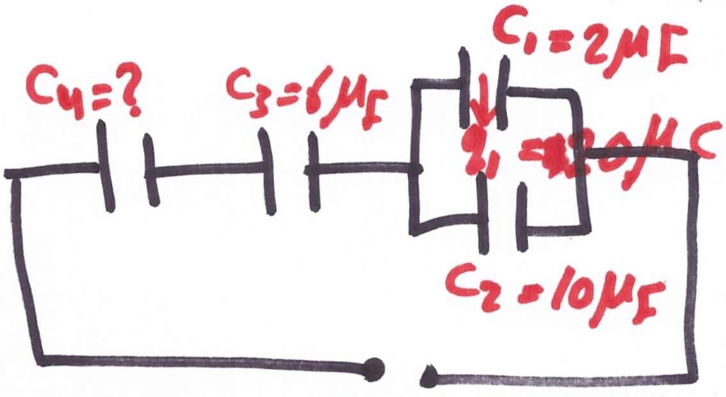
③ $U_3 = \frac{1}{2} q_3 V_3 = \frac{1}{2} \times 160\mu \times 40 = 3200\mu$ [illegible]

Ex:



$V_{ab} = 120V$

Find: C_{eq}, V, q for all C's, U_3.



$\rightarrow q_1 = 20\mu C$

$V = 80V$

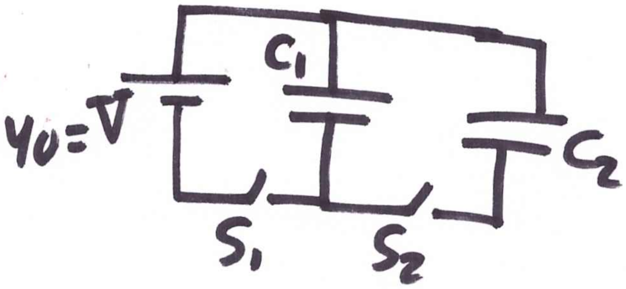
Find C_4.

q, V for all C's.

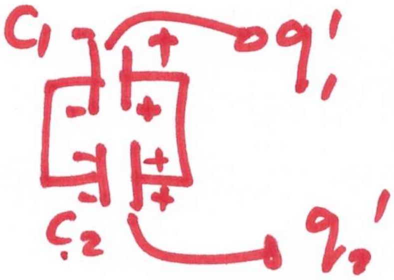
Ex: Capacitor has $C_1 = 6\mu F$, Connected to Voltage of 40 Volt. then disconnected from battary, and then Connected to anothe Capaciter $C_2 = 3\mu F$ initially Uncharged. Find $\underline{q}$ and $\underline{V}$ for C_1 and C_2 after Connected togather.

Sol:

$$\left.\begin{array}{l} C_1 = 6\mu F \\ V_1 = 40 \text{ Volt} \end{array}\right\} \Rightarrow q_1 = 240\mu C$$



$$\left.\begin{array}{l} C_2 = 3\mu F \\ V_2 = 0 \end{array}\right\} \Rightarrow q_2 = 0$$



$$V_1' = V_2'$$

$$\frac{q_1'}{{}_2\cancel{6\mu}} = \frac{q_2'}{\cancel{3\mu}}$$

$$\Rightarrow \boxed{q_1' = 2q_2'}$$

$$\sum_{bef} q = \sum_{after} q'$$

$$q_1 + q_2 = q_1' + q_2'$$

$$240\mu + 0 = q_1' + q_2'$$

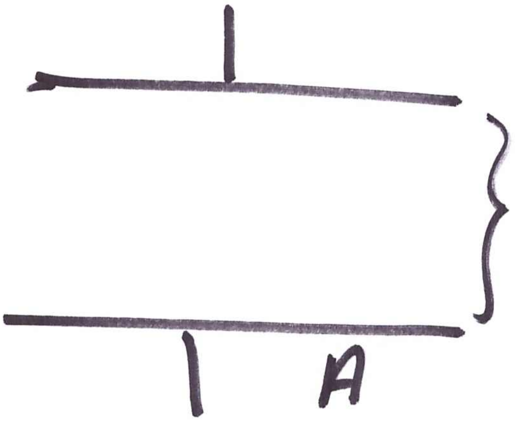
$$\Rightarrow 240\mu = 2q_2' + q_2' = 5q_2'$$

$$\Rightarrow \boxed{q_2' = 80\mu C}$$

$$\Rightarrow \boxed{q_1' = 160\mu C}$$

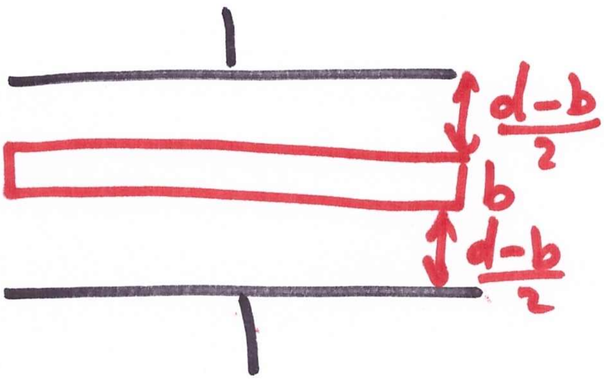
$$V_1' = \frac{160\mu}{6\mu} = 26.7\,V = V_2'$$

metalic slap inside C



$$\Rightarrow C = \frac{\epsilon_0 A}{d}$$

$\Downarrow$

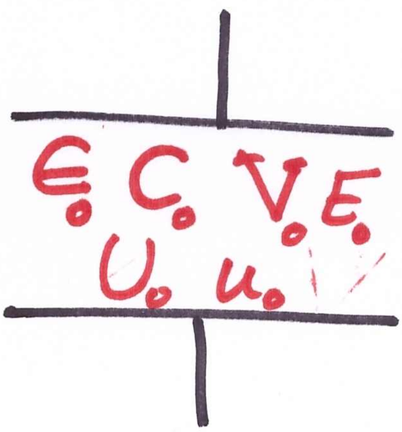


$$\Rightarrow C = \frac{\epsilon_0 A}{(d-b)}$$

Insulatic material

(Dielectric material)

K : dielectric constant (K = 1 air

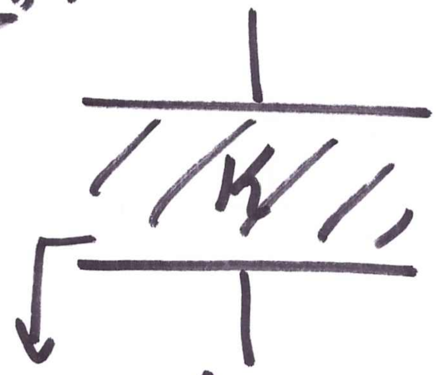


$\Rightarrow$

$\boxed{\epsilon = K\epsilon_0}$ $\boxed{C = KC_0}$

$\boxed{V = \frac{V_0}{K}}$ $\boxed{E = \frac{E_0}{K}}$ $\boxed{U = \frac{U_0}{K}}$ $\boxed{u = \frac{u_0}{K}}$

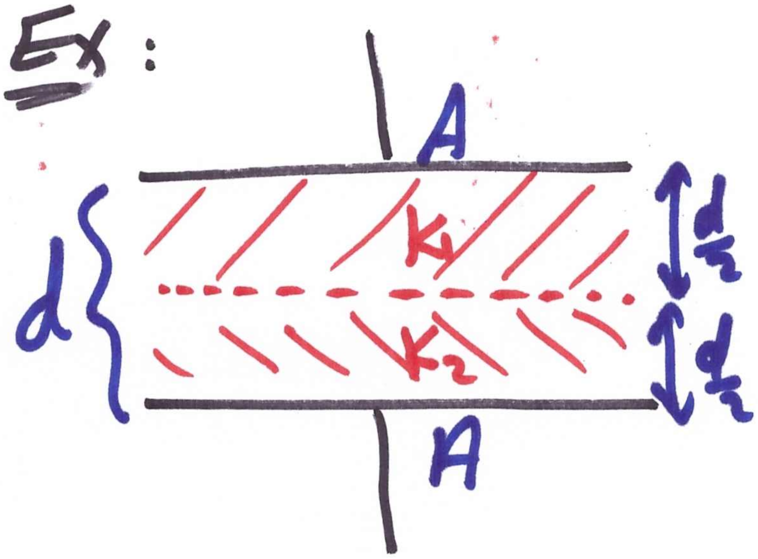
Ex:



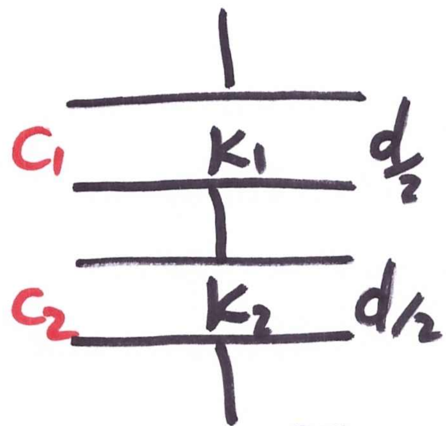
$$\Rightarrow C = \frac{K \epsilon_0 A}{d} = K C_0$$

$$C_0 = \frac{\epsilon_0 A}{d}$$

Ex:



$\Rightarrow$



$$C_1 = \frac{2K_1 \epsilon_0 A}{d} \quad C_0$$

$$C_2 = \frac{2K_2 \epsilon_0 A}{d} \quad C_0$$

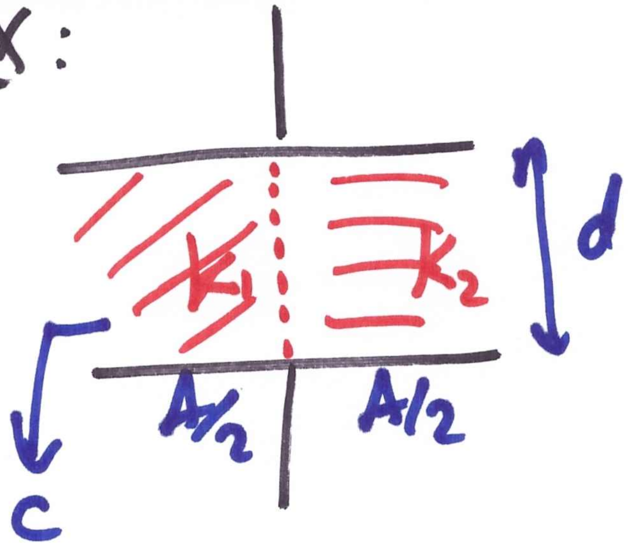
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{d}{2K_1 \epsilon_0 A} + \frac{d}{2K_2 \epsilon_0 A}$$

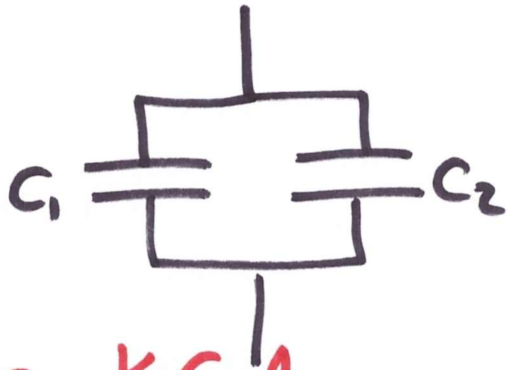
$$\frac{1}{C_{eq}} = \frac{dK_2 + dK_1}{2K_1K_2 \epsilon_0 A} \Rightarrow C_{eq} = \frac{2K_1K_2}{(K_1+K_2)} \frac{\epsilon_0 A}{d}$$

$$\boxed{C_{eq} = \left(\frac{2K_1K_2}{K_1+K_2}\right) C_0}$$

Ex:



$\Rightarrow$



$$C_1 = \frac{K_1 \epsilon_0 A}{d\ 2}$$

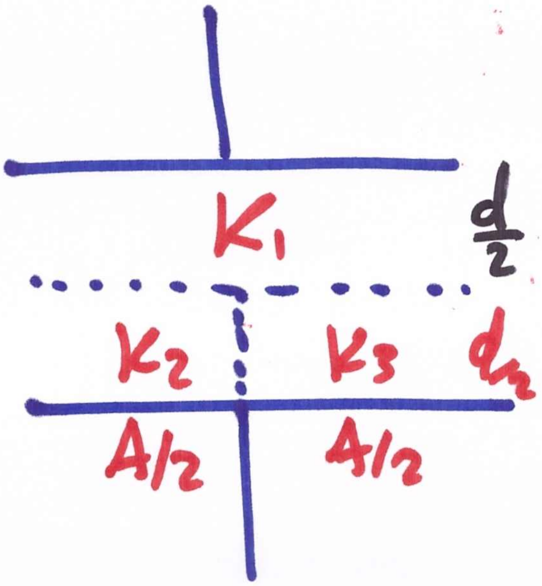
$$C_2 = \frac{K_2 \epsilon_0 A}{2d}$$

$$C_{eq} = C_1 + C_2$$

$$= \frac{K_1 \epsilon_0 A}{2d} + \frac{K_2 \epsilon_0 A}{2d}$$

$$= \left(\frac{K_1 + K_2}{2}\right)\frac{\epsilon_0 A}{d}$$

$$C_{eq} = \frac{K_1 + K_2}{2} C_0$$

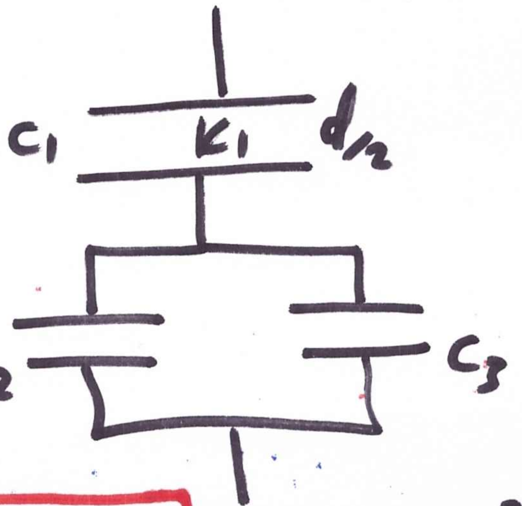


$$C_1 = \frac{K_1 \epsilon_0 A}{d/2}$$

$$C_2 = \frac{K_2 \epsilon_0 \frac{A}{2}}{\cancel{2}/d_{/\cancel{2}}}$$

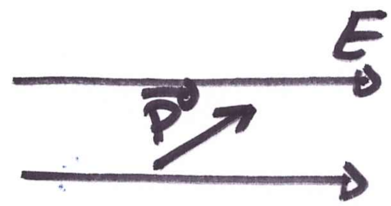
$$= \frac{K_2 \epsilon_0 A}{d}$$

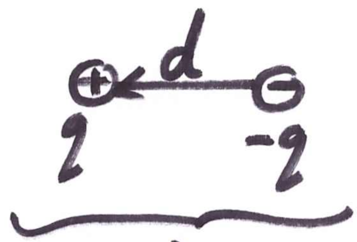
$$C_3 = \frac{K_3 \epsilon_0 A}{d}$$



cH: 26

Dipole → inside E





$$\vec{P} = q\vec{d}$$

$$P = qd$$

* $\vec{\tau} = \vec{P} \times \vec{E}$

$$\tau = PE \sin\theta_{PE}$$

* $U = -\vec{P} \cdot \vec{E}$

$$= -PE\cos\theta_{PE}$$

$$U_{max} = PE \quad (\theta = 180°)$$

$$U_{min} = -PE \quad (\theta = 0)$$

$$\Delta U = 2PE$$

Q51 $q = ✓$ $q = (-1.2, 1.1)$ $\vec{E} = (\ )\hat{i} - (\ )\hat{j}$

$-q = (1.4, -1.3)$

a) $\vec{P} = q\vec{d}$

$= q(-2.6\hat{i} + 2.4\hat{j})$

b) $\vec{\tau} = \vec{P} \times \vec{E}$

$\vec{\tau} \perp \vec{P}, \vec{E}$

c) $U = -\vec{P} \cdot \vec{E}$

$= \ \ J$

d) $\Delta U = 2PE$

$$I_{av} = \frac{\Delta q}{\Delta t} \Rightarrow \Delta q = I \Delta t$$

$$I_{ins} = \frac{dq}{dt} \Rightarrow \Delta q = \int_{t_1}^{t_2} I\,dt$$

Ex: If $q = 4t^2 - 5t + 1$, Find the Current at $t = 2$ sec.

$$\Rightarrow I_{ins} = \frac{dq}{dt} = 8t - 5 \Rightarrow I = 11\ \cancel{sec}\ \text{Amp.}$$

$$I_{av} = \frac{\Delta q}{\Delta t} = \frac{q_2 - q_1}{t_2 - t_1} \qquad t = 0 \rightarrow 2.$$

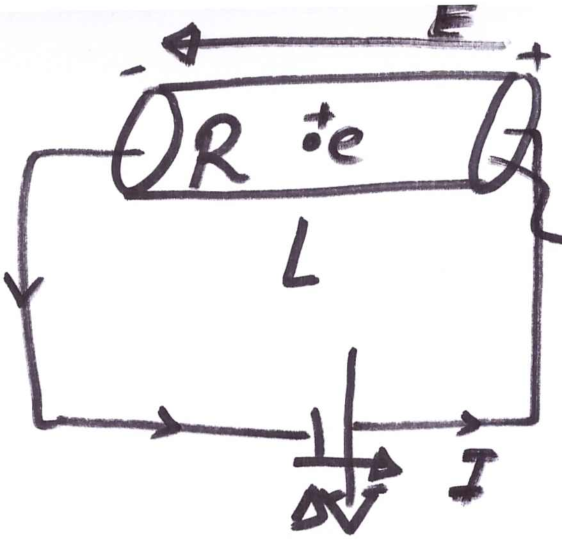
$$q_1(t=0) = 1$$

$$q_2(t=2) = 4(4) - 5(2) + 1 = 7$$

$$\Rightarrow I = \frac{7-1}{2-0} = \frac{6}{2} = 3\ A.$$

Ex: $I = 6t - 2$, Find charge from $t=0$ to $t=1$.

$$\Delta q = \int_0^1 (6t-2)\,dt = 3t^2 - 2t \Big|_0^1 = 3 - 2 = 1\ \text{Col.}$$



$A = \pi r^2$ ($r \equiv$ radius)

$Vol = AL$

$q = I * t$

$$\boxed{\Delta V_R = I * R}$$ Ohm's law.

$P_R = IV = I^2 R = \frac{V^2}{R}$

$U = P * t$

$I = \frac{q}{t}$

$q = ne$

$I = n' e v_d A$

$v_d = \frac{L}{t}$

$J = \frac{I}{A} = n' e v_d$

$J = \sigma E$

$R = \frac{\rho L}{A}$

$\rho = \frac{1}{\sigma}$

$\Delta V_R = E * L$

$F = eE$

$\sigma = \frac{n' e^2 \tau}{m_e}$

q = charge inside the resistance.

I = Current

t = time of current moving

ΔV = Potential (voltage) difference accross the $\underline{\underline{R}}$.

n = number of electrons pass through the wire.

e = electron charge $(e = +1.6 * 10^{-19} C)$

n' = electron's density (no of $\underline{\underline{e}}$ in Unit Volume) $[e/m^3]$

v_d = drift Velocity (سرعة الانسياق)

A = Area of the wire. (m^2)

L = length of the wire.

J = Current density $[A/m^2]$

R = Resistance $(\text{ohm} = \Omega)$

ρ = resistivity $(\Omega \cdot m)$

σ = Conductivity $(\Omega \cdot m)^{-1}$

E = Electric field.

P = Power (energy rate) $\Rightarrow$ watt

U = energy in $\underline{\underline{R}}$.

τ = av. time interval between two Successive collisions.

m_e = mass of $\underline{e}$

Ex: Resistance of length 10m and radius of
(L)

$\left(\frac{2}{\sqrt{\pi}}\,cm\right)$, has a conductivity of $4 * 10^{6} (\Omega \cdot m)^{-1}$,
$r \rightarrow A = 4 * 10^{-4} m^2$ $\qquad \sigma$

Connected accross a pot. diff of 20-V for
ΔV

10-sec, Find:
t

1) resistivity 2) resistance 3) Current

4) charge 5) no of e passes through the wire. (n)

6) drift velocity 7) no of e per unit vol. (n')

8) Current density 9) Electric Field

10) Power 11) energy 12) Force on e.

13) time between 2 successive Collisions (τ)

$$\begin{tikzcd} \underline{\underline{R}} \arrow[r] \arrow[dr] & L & \\ & A \arrow[r] & r \end{tikzcd}$$

$$\begin{tikzcd} r \arrow[r, "\text{تربيع}"] & A \arrow[r, "\text{مقلوب}"] \arrow[l, bend left, "\text{الجذر}"] & L \arrow[r, "\text{تربيع}"] \arrow[l, bend left, "\text{مقلوب}"] & R \arrow[l, bend left, "\text{الجذر}"] \end{tikzcd}$$

$r_2 = 2r_1 \Longrightarrow A_2 = 4A_1 \Rightarrow L_2 = \frac{1}{4} L_1 \Rightarrow R_2 = \frac{1}{16} R_1$

$A_2 = 2A_1 \longrightarrow L_2 = \frac{1}{2} L_1 \longrightarrow R_2 = \frac{1}{4} R_1$

$R_2 = 3R_1 \Longrightarrow L_2 = \sqrt{3}\, L_1 \Rightarrow A_2 = \frac{1}{\sqrt{3}} A_1 \Rightarrow r_2 = \frac{1}{3^{1/4}} r_1$

$\Downarrow$

$\left(\frac{1}{3}\right)^{\frac{1}{4}}$

$* \; r_2 = 2r_1$

$A_1 = \pi r_1^2$

$A_2 = \pi r_2^2$

$= \pi (2r_1)^2$

$= 4\pi r_1^2$

$A_2 = 4A_1$

$V_1 = V_2$

$L_1 A_1 = L_2 A_2$

$L_1 \cancel{A_1} = L_2 4\cancel{A_1}$

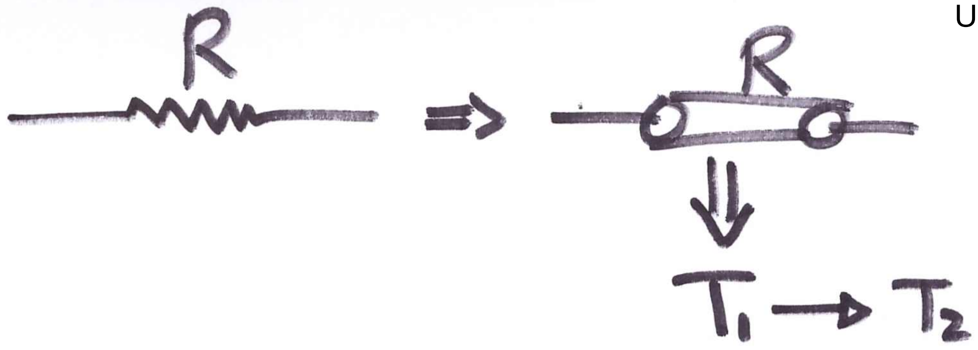
$L_2 = \frac{1}{4} L_1$

$R_1 = \frac{\rho L_1}{A_1}$

$R_2 = \frac{\rho L_2}{A_2}$

$= \frac{\rho L_1}{4 \times 4A_1}$

$= \frac{\rho L_1}{A} * \frac{1}{16} = \underline{R_1}$



R_0 : orginal resistance.

R : new resistance.

α : Temp. Coeff. of resistance. $\left(\frac{1}{C^\circ}\right)$

$$R = R_0[1 + \alpha(T_f - T_i)]$$

$$\rho = \rho_0[1 + \alpha(T_f - T_i)]$$

Ex : $\alpha = 2 * 10^{-5}/c$

$T_i = 50°c$ $\qquad$ $T_f = ??$

$R_0 = 10\,\Omega \longrightarrow R = 20\,\Omega$

$$20 = 10[1 + 2*10^{-5}(T_f - 50)]$$

$T_f = 50050\,C°$.

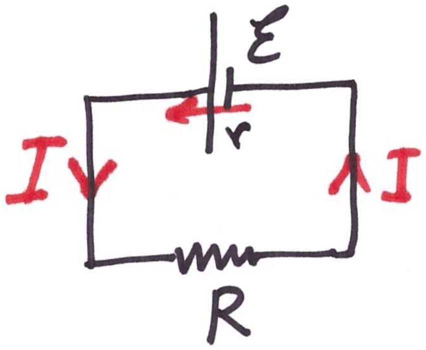
$\frac{\Delta R}{R} \Rightarrow R = R_0 + R_0\alpha(\Delta T)$ | $\frac{\Delta R}{R}$: fractional change in R

$\frac{\Delta R}{R} = \alpha\,\Delta T$

1

CH:28 Electrical Circuits

الدارات الكهربائية



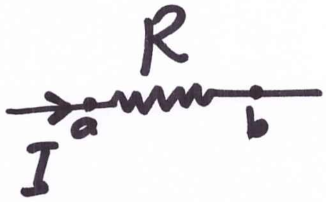
I: Current (A)

R: external resistance (Ω)

r: internal resistance (Ω)

$\mathcal{E}$: electromotive force (V)

Resistance :



$\Rightarrow$ $\boxed{V_{ab\,(R)} = I R}$ — ohm's law.

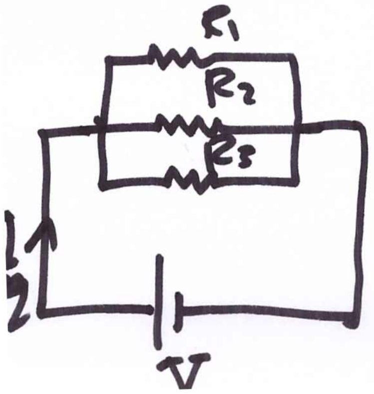
$$P_R = I * V_R = I^2 R = \frac{V^2}{R}.$$

$$U = P * t$$

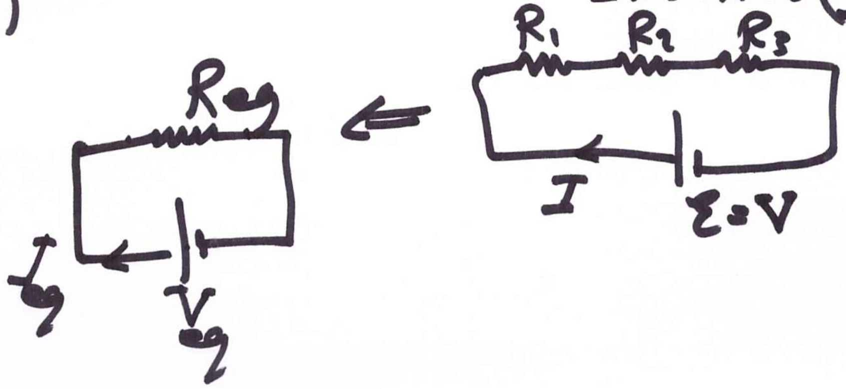
Resistors' Connection

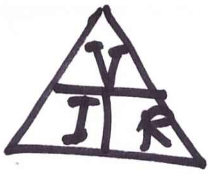
توصيل المقاومات

In Parallel (التوازي)



In Series (التوالي)





التوالي

التيار متساوي

$$\boxed{I_{eq} = I_1 = I_2 = I_3 = \dots}$$

الجهد يتوزع

$$V_{\text{بطارية}} = V_{eq} = V_1 + V_2 + V_3 + \dots$$

المقاومة مجموعة

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \dots}$$

التوازي

التيار يتوزع :

$$I_{eq} = I_1 + I_2 + I_3 + \dots$$

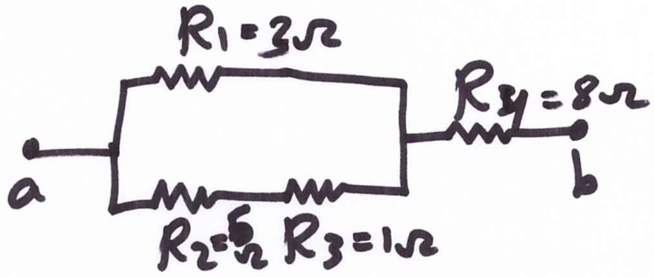
الجهد متساوي :

$$\boxed{V_{eq} = V_1 = V_2 = V_3 = \dots}$$

المقاومة مقلوبة :

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

Ex: Find the equivalent resistance (R_{eq}):



$R_2, R_3 \xrightarrow{\text{توالي}} R_5$

$R_1, R_5 \xrightarrow{\text{توازي}} R_6$

$R_4, R_6 \xrightarrow{\text{توالي}} R_7 = R_{eq}$.

$$* \; R_5 = R_2 + R_3 = 5 + 1 = 6\,\Omega$$

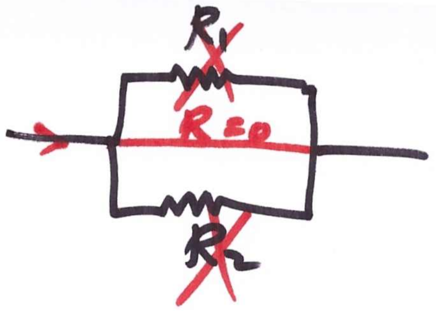
$$\frac{1}{R_6} = \frac{1}{R_5} + \frac{1}{R_1}$$

$$\frac{1}{R_6} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

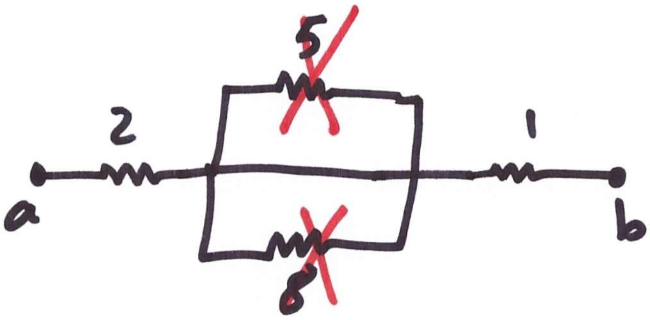
$$\boxed{R_6 = 2\,\Omega}$$

$$R_7 = R_4 + R_6 = 8 + 2$$

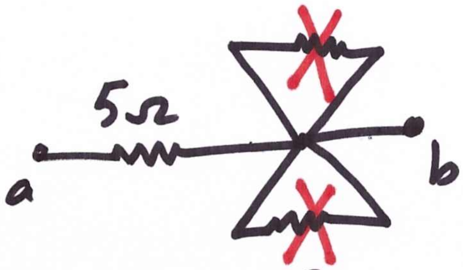
$$\boxed{R_7 = 10\,\Omega}$$



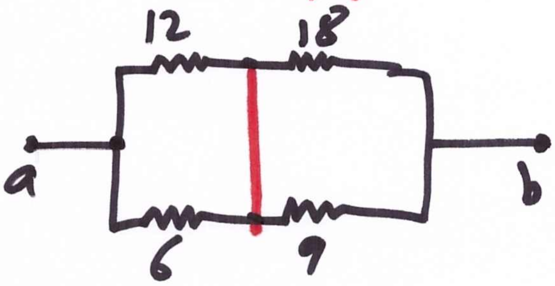
$\Rightarrow R_{eq} = 0$

* 

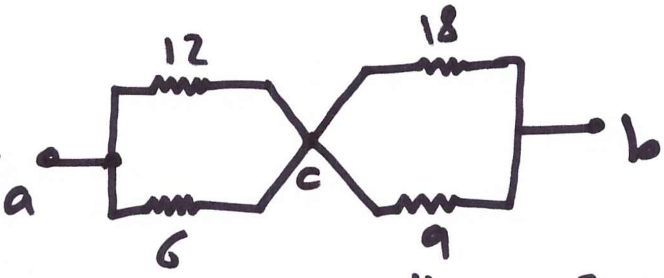
$R_{ab} = 3\Omega$.

* 

$R_{ab} = 5\Omega$

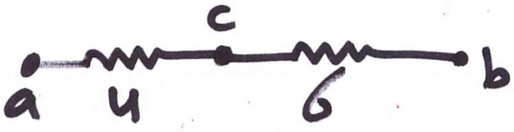
* 

$\Rightarrow$



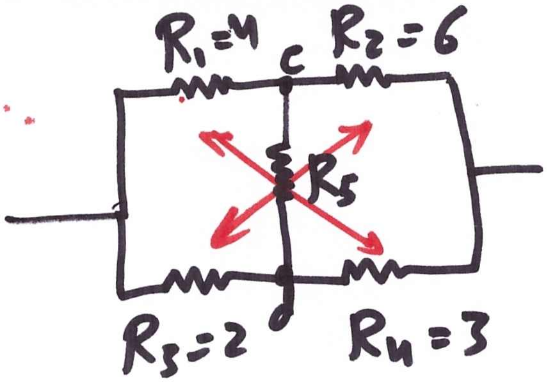
⇓ توازي

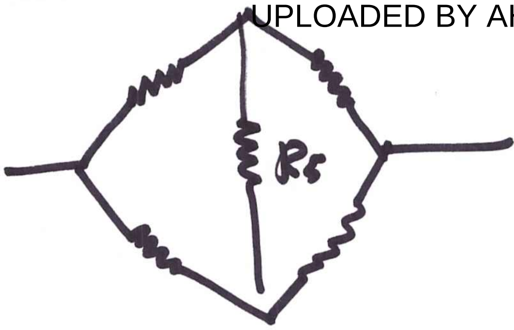
توازي ⇓

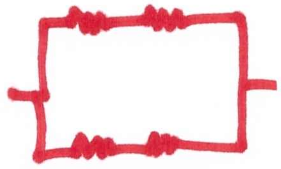


⇓

$R_{ab} = 10\Omega$.





$\underbrace{R_1 * R_4 \overset{?}{=} R_2 * R_3}$ $\Longrightarrow$ R_5 تلغى $\Longrightarrow$ 

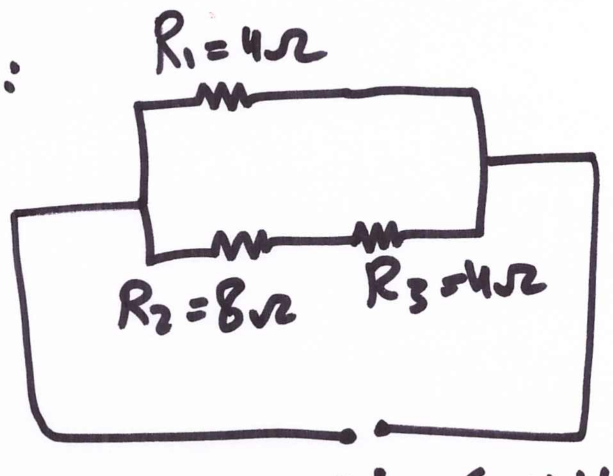
$\Downarrow$

$I_{R_5} = 0$

$\Downarrow$

$\boxed{V_c = V_d}$ or $\boxed{V_{cd} = 0}$

Ex:



Find:
1) Req.
2) Voltage and Current in each resistor.
3) Power in R_3

١) نرسم خريطة مع تغيير القطع الإضافي.

٢) نجد Req

٣) نجد جميع المعلومات عن القطع الإضافي

٤) نجد بقية [illegible]

$R_2, R_3 \xrightarrow{\text{توالي}} R_4$

$R_1, R_4 \xrightarrow{\text{توازي}} R_5 = R_{eq}$

$V_5 = 60$ Volt

1

$$R_4 = R_2 + R_3$$
$$= 8 + 4 = 12\,\Omega$$

$$\frac{1}{R_5} = \frac{1}{R_1} + \frac{1}{R_4}$$
$$= \frac{1}{4} + \frac{1}{12}$$
$$\boxed{R_5 = 3\,\Omega}$$

2

$$V_5 = 60\text{ V}$$
$$R_5 = 3\,\Omega$$
$$\Rightarrow I_5 = \frac{V_5}{R_5} = \frac{60}{3} = 20\text{ A}.$$

$$V_5 = V_4 = V_1 = 60\text{ V}$$
$$\rightarrow I_1 = \frac{V_1}{R_1} = \frac{60}{4} = 15\text{A}$$
$$\rightarrow I_4 = \frac{V_4}{R_4} = \frac{60}{12} = 5\text{A}$$

$$I_4 = I_2 = I_3 = 5\text{A}$$
$$\rightarrow V_3 = I_3 R_3 = 5 * 4 = 20 \text{ Volt}$$
$$\rightarrow V_2 = I_2 R_2 = 5 * 8 = 40 \text{ Volt}.$$

3

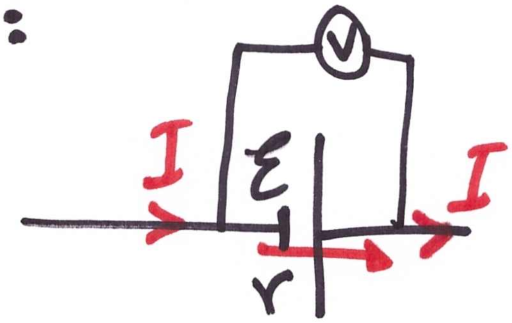
$$P_3 = I_3^2 R_3$$
$$= (5)^2 * 4$$
$$= 25 * 4$$
$$= 100 \text{ watt}$$

$$P_1 = I_1^2 R_1$$
$$= (15)^2 * 4$$
$$= 900 \text{ watt}$$

$$P_2 = I_2^2 R_2$$
$$= (5)^2 * 8$$
$$= 200 \text{ watt}$$

$$P_5 = I_5^2 R_5$$
$$= (20)^2 * 3$$
$$= 400 * 3$$
$$= 1200 \text{ watt}$$

Battery :



* V_ε : Potential difference accross the battery.

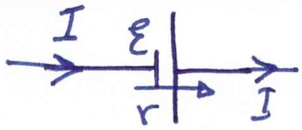
I with ε

$$\boxed{V_\varepsilon = \varepsilon \mp Ir}$$

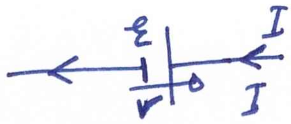
I against ε

$V_\varepsilon = \varepsilon$ if

r = 0	ideal battery
I = 0	open circuit



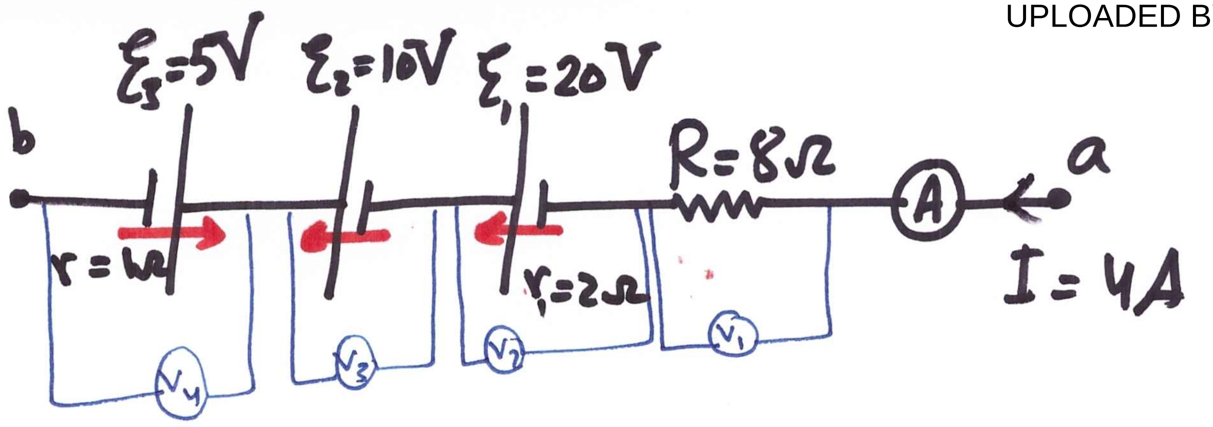
⇒ ⊖ discharging



⇒ ⊕ charging

* Power of battery: $\boxed{P_\varepsilon = I\varepsilon}$

* energy (U) ⇒ $U = P * t$.



What are the readings of Ⓐ and Ⓥ's.

ملخص للقوانين

Ⓥ - $\boxed{V}$

$\boxed{\varepsilon}$

$$V_\varepsilon = \varepsilon \mp IR$$

$\boxed{R}$

$$V = IR$$

(watt) ≡ Power [$U = P * t$]

$\boxed{\varepsilon}$

$$P = I\varepsilon$$

$\boxed{R}$

$$P_R = I^2R = \frac{V^2}{R} = IV$$

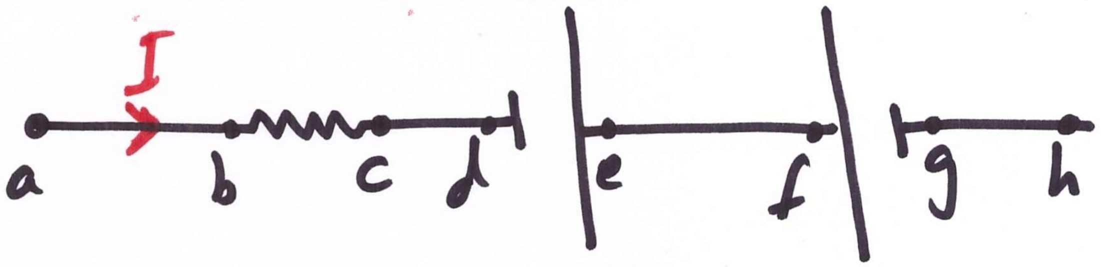
$A = I = 4A.$

$V_1 = V_R = IR = 4 * 8 = 32$ Volt.

$V_2 = V_{\mathcal{E}_1} = \mathcal{E}_1 - Ir_1$
$= 20 - 4 * 2$
$= 12$ Volt.

$V_3 = V_{\mathcal{E}_2} = \mathcal{E} = 10$ Volt.

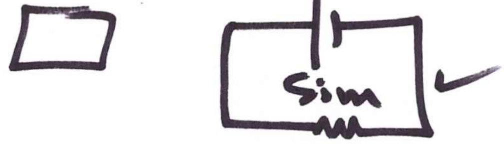
$V_4 = V_{\mathcal{E}_3} = \mathcal{E}_3 + Ir_3$
$= 5 + 4 * 1$
$= 9$ Volt.

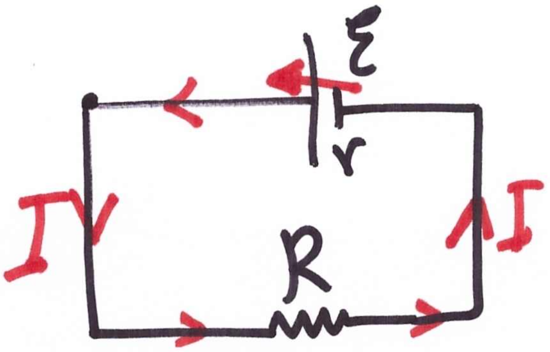


$I_a = I_b = I_c = I_d = I_e = I_f = I_g = I_h$

$V_a = V_b > V_c = V_d < V_e = V_f > V_g = V_h$

Simple Circuits:

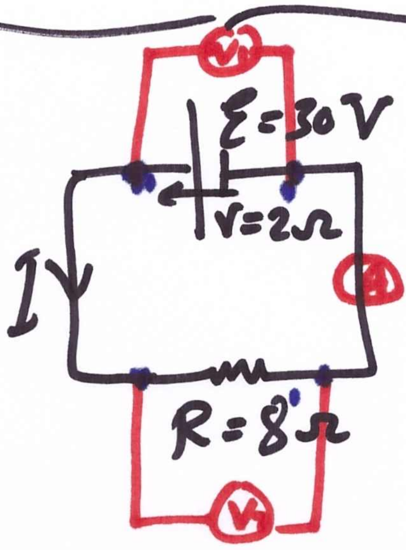




$\Rightarrow \sum \varepsilon = I \sum R$

R or r

Ex:



Find:
1) Reading of (A).
2) " " V_1, V_2
3) Power disappated in R
4) " " " r.
5) Produced Power by ε
6) Consumed energy in R through 2-min.

1) $\sum \varepsilon = I \sum R$

$30 = I(2+8)$

$I = 3A \equiv$ (A)

2) $V_\varepsilon = \varepsilon - Ir$

$= 30 - 3 * 2$

$= 24$ Volt

$V_2 = V_R = IR = 3 * 8$

$= 24$ Volt.

③ $P_R = I^2 R = (3)^2 * 8 = 72$ watt.

④ $P_r = I^2 r = (3)^2 * 2 = 18$ watt

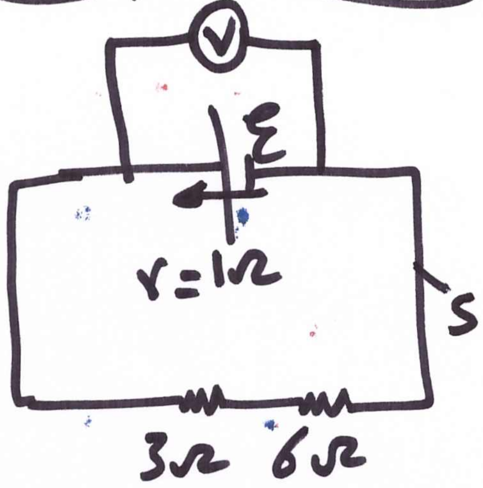
⑤ $P_\varepsilon = I\varepsilon = 3 * 30 = 90$ watt.

⑥ $U_R = P_R * t = 72 * 120$

$= 8640$ J.

⑦ drop in ε Potential :⟹ $I * r = 3 * 2 = 6$ Volt

Ex:



reading of Ⓥ is 40V when S is open. what is the reading if the $\underline{S}$ is closed.

S is open:- $I = 0 \Rightarrow$ Ⓥ $= V_\varepsilon = \varepsilon - \cancel{Ir}$

$\boxed{40 = \varepsilon}$

S is closed: Ⓥ $= V_\varepsilon = \varepsilon - Ir$

$= 40 - 4 * 1$

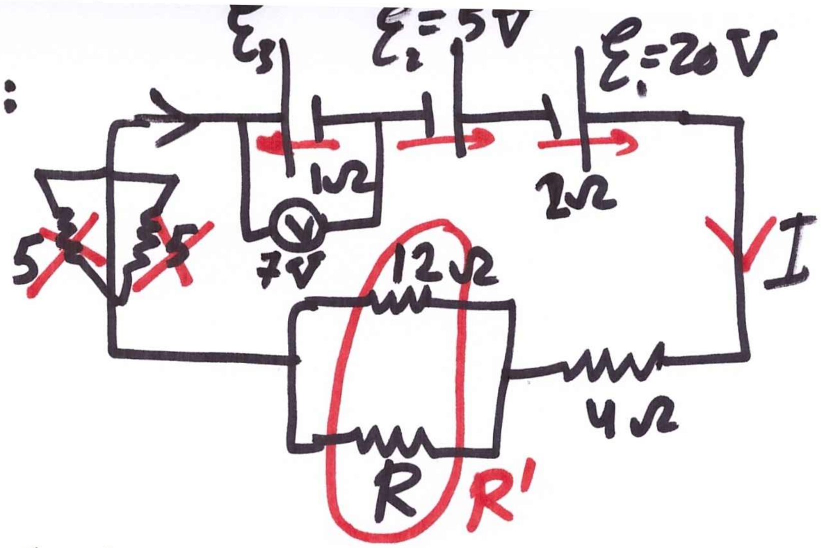
$= 36$ Volt.

$\Sigma\varepsilon = I\Sigma R$

$40 = I(1+3+6)$

$I = 4A$.

Ex :



* Power in $R = 4\Omega$ is 16 watt.

Find ① I ② ε_3 ③ R

✓ ε x
R · x
r ✓
✓ I x

$$P_R = I^2 R$$
$$16 = I^2 * 4$$
$$I^2 = 4$$
$$\boxed{I = 2A}$$

$$V_{\varepsilon_3} = \varepsilon_3 + Ir$$
$$7 = \varepsilon_3 + 2*1$$
$$\boxed{\varepsilon_3 = 5 \text{ volt}}$$

$$\Sigma\varepsilon = I\Sigma R$$
$$20+5-5 = 2(2+1+R'+4)$$
$$20 = 2(7+R')$$
$$10 = 7+R'$$
$$\boxed{R' = 3\Omega}$$

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{12}$$
$$\frac{1}{3} - \frac{1}{12} = \frac{1}{R}$$
$$\boxed{R = 4\Omega}$$

Find current in R.
R. $\parallel$ توازي → R'

$\Rightarrow V_{R'} = IR'$
$= 2*3$

$V_R = 6$ Volt
$\Rightarrow I' = \frac{V_R}{R} = \frac{6}{4} = 1.5A$

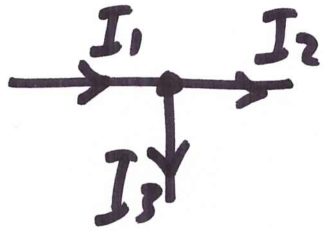
الدارات المعقدة (أكثر من حلقة)

قانونا كيرتشوف

خطوات حل الدارات المعقدة :-

١) نحدد اتجاه التيارات إذا لم تكن محددة (بشكل عشوائي)

٢) نطبق في قانون كيرتشوف الأول :



$$\Sigma I_{in} = \Sigma I_{out}$$

* نستفيد من المعطيات الإضافية إن وجدت .

V_{ab} , V , P_ε , P_R .

٣) نحدد مساراً مغلقاً لنا نتحرك عليه.

٤) نطبق في قانون كيرتشوف الثاني :

$(V_{ab}=V_a-V_b)$ $$V_{ab} = \Sigma IR - \Sigma \varepsilon$$

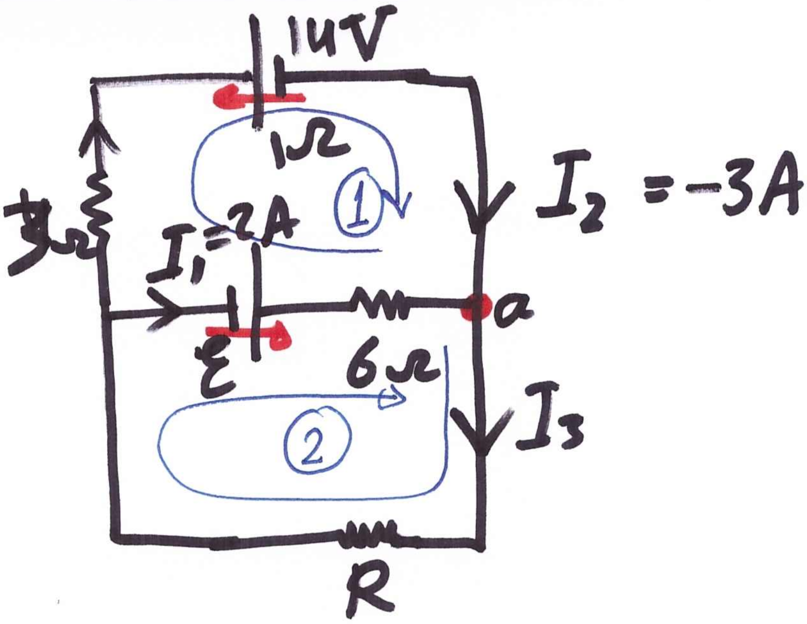
معه : +
عكسه : −

* إذا وجدت I سالبة ← اتجاه I هو عكس الاتجاه المفروض .

* $V_{aa} = V_{bb} = 0$

* إذا تغير أي شيء في الدارة فتصبح مسألة جديدة.

Ex:



Find:

1) I_3
2) ε
3) R

$$\sum I_{in} = \sum I_{out}$$

$$I_1 + I_2 = I_3$$

$$2 + -3 = I_3$$

$$\boxed{I_3 = -1A}$$

$$V_{aa} = (-6*2 + 3*I_2 + 1*I_2) - (-\varepsilon - 14)$$

$$0 = -12 + 4*(-3) + \varepsilon + 14$$

$$-\varepsilon = -12 - 12 + 14$$

$$-\varepsilon = -10 \Rightarrow \boxed{\varepsilon = 10 \text{ Volt}}$$

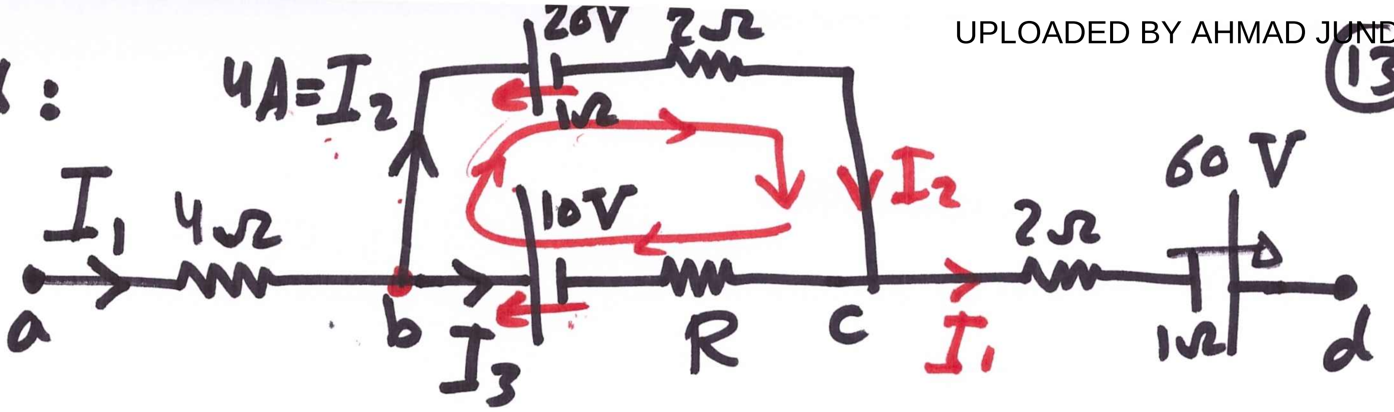
$$V_{aa} = (R*I_3 + 6I_1) - (\varepsilon)$$

$$0 = -R + 6*2 - 10$$

$$R = 12 - 10$$

$$\boxed{R = 2\Omega}$$

Ex:



If $\underrightarrow{V_{ab}} = 40\ V$, Find: 1) I_3, 2) R

3) V_{dc}

Sol: (1)

$$\Sigma I_{in} = \Sigma I_{out}$$

$$I_1 = I_2 + I_3$$

$$\boxed{I_1 = 4 + I_3}$$

$$V_{ab} = (\Sigma IR) - (\Sigma \varepsilon)$$

$$40 = (4I_1) - (0)$$

$$\boxed{I_1 = 10\ A} \Longrightarrow \boxed{I_3 = 6A}$$

(3)

$$\underrightarrow{V_{dc}} = (\bar{3} \times 10) - (\bar{6}0)$$

$$= -30 + 60$$

$$= 30\ \text{Volt}.$$

(2)

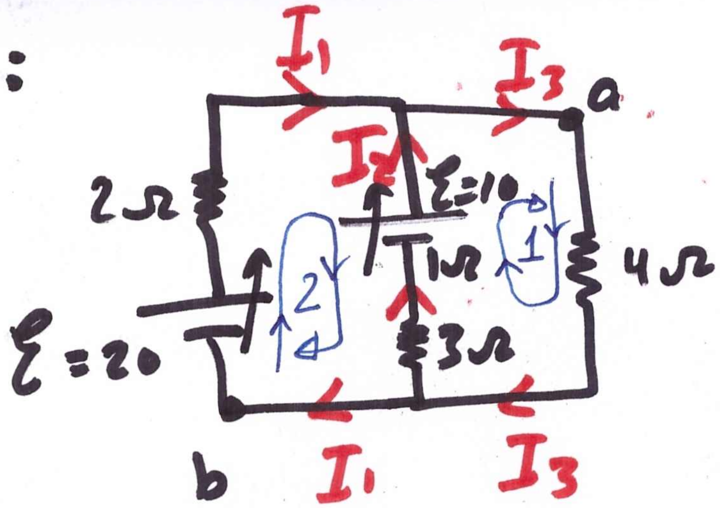
$$V_{cc} = (-6 * R + 3 * 4) - (10 - 20)$$

$$0 = -6R + 12 - 10 + 20$$

$$6R = 22$$

$$\boxed{R = 3.7\ \Omega}$$

Ex:



Find:

1) each current.
2) V_{ab}
3) if $V_b = 10V$, find V_a.

$\sum I_{in} = \sum I_{out}$

$$\boxed{I_1 + I_2 = I_3}$$

$$V_{aa} = (4I_3 + 4I_2 \quad) - (10 \quad)$$

$$0 = 4I_3 + 4I_2 - 10$$

$$10 = 4(I_1 + I_2) + 4I_2$$

$$\boxed{10 = 4I_1 + 8I_2} \text{ —— } ①$$

$$V_{bb} = (2I_1 - 4I_2 \quad) - (20 - 10 \quad)$$

$$0 = 2I_1 - 4I_2 - 10$$

$$\boxed{10 = 2I_1 - 4I_2} \text{ —— } ②$$

$\Rightarrow$ $I_1 = 3.75\,A$

$\boxed{I_2 = -0.625}$

$\Rightarrow I_3 = 3.75 + (-0.625)$

$\boxed{I_3 = 3.125\,A}$

~~Ex~~1. 2 $V_{ab} = (4 * I_3) - (0)$

$b \longleftarrow a$

$= 4 * 3.125$

$= 12.5$ Volt.

$V_{ab} = (2 * -3.75) - (-20)$

$= -7.5 + 20$

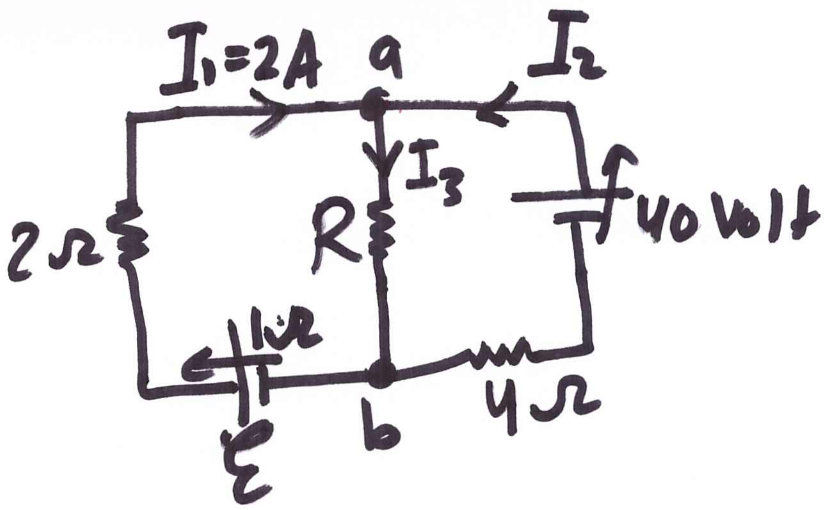
$= 12.5$ Volt.

3 $V_{ab} = V_a - V_b$

$12.5 = V_a - 10$

$\boxed{V_a = 22.5 \text{ Volt}}$

Ex:



Find
1) all I's.
2) R
3) $\mathcal{E}$

where $V_{ab} = 24V$.

Sol

$$\sum I_{in} = \sum I_{out}$$

$$I_1 + I_2 = I_3$$

$$\boxed{2 + I_2 = I_3}$$

$$V_{\underrightarrow{ab}} = (-4 * I_2) - (-40)$$

$$24 = -4 I_2 + 40$$

$$-16 = -4 I_2$$

$$\boxed{I_2 = 4A} \Rightarrow \boxed{I_3 = 6A}$$

$$V_{ab} = (R * 6) - (0)$$

$$24 = 6R$$

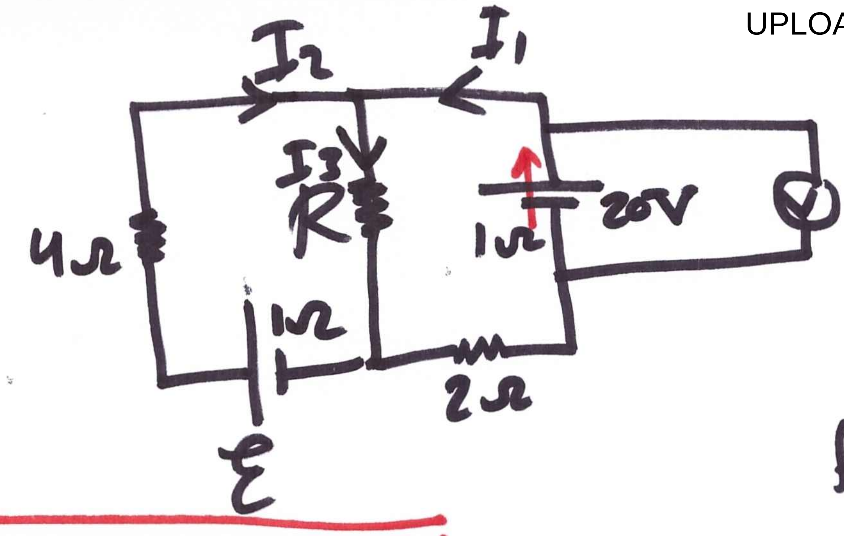
$$\boxed{R = 4\Omega}$$

$$V_{\underrightarrow{ab}} = (-3 * 2) - (-\mathcal{E})$$

$$24 = -6 + \mathcal{E}$$

$$\boxed{\mathcal{E} = 30\,V}$$

Ex:



$V = 14\,V$

$P_{4\Omega} = 64$ watt

find: I's, R, ε

$$\boxed{I_1 + I_2 = I_3}$$

$P_{4\Omega} = I_2^2 R$

$64 = I_2^2\, 4$

$I_2^2 = 16$

$\boxed{I_2 = 4A}$

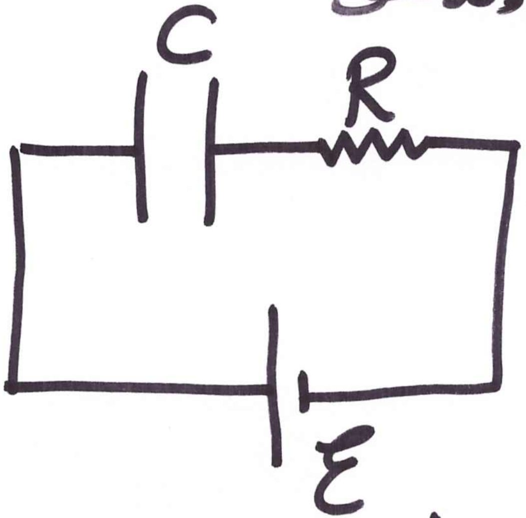
$V_\varepsilon = \varepsilon - I_1 r$

$14 = 20 - I_1 * 1$

$\boxed{I_1 = 6A} \Longrightarrow \boxed{I_3 = 10A}$

RC - circuit

دارة مقاومة ومكثف



$$\mathcal{E} = V_R + V_c$$

$$\mathcal{E} = IR + \frac{q}{c}$$

	$t=0$	$t=\infty$ long time
I	I_{max}	0
q	0	q_{max}

$$I_{max} = \frac{\mathcal{E}}{R}$$

$$q_{max} = C\mathcal{E}$$

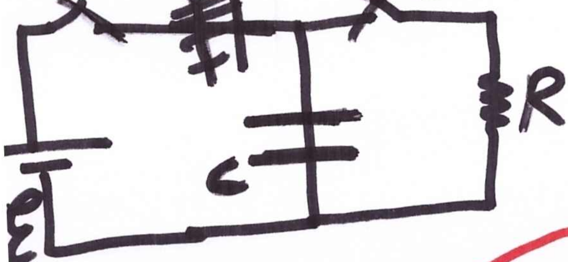
τ : Time Constant

ثابت الزمن يساوي

$$\tau = RC$$

$$I = I_{max}\, e^{-t/RC}$$

$$q = q_{max}\left(1 - e^{-t/RC}\right)$$



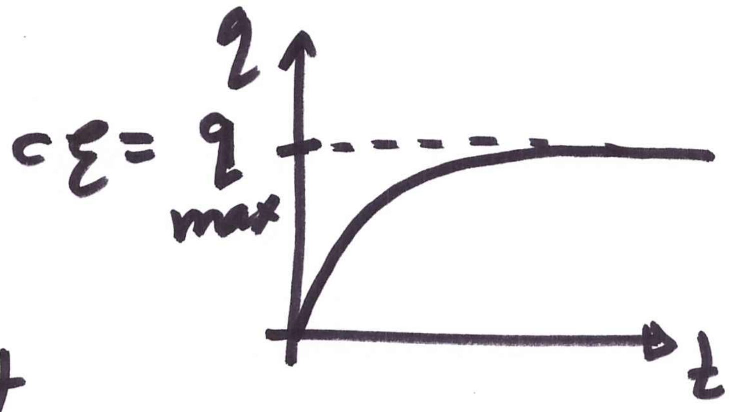
discharging

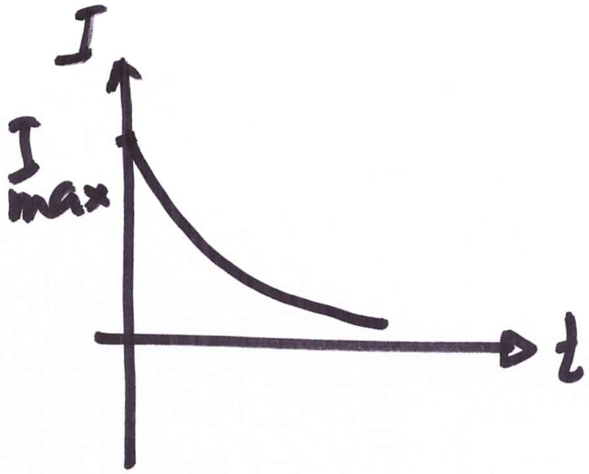
	$t=0$	$t=\infty$
I	I_{max}	0
q	q_{max}	0

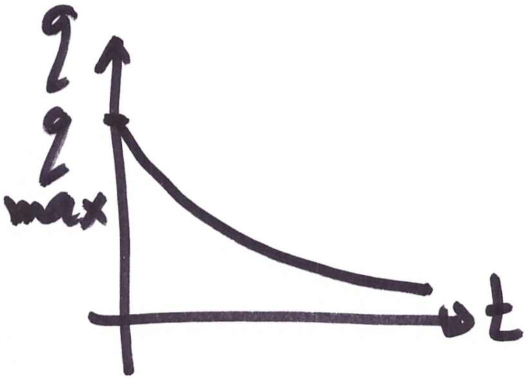
$$I = I_{max}\, e^{-t/RC}$$

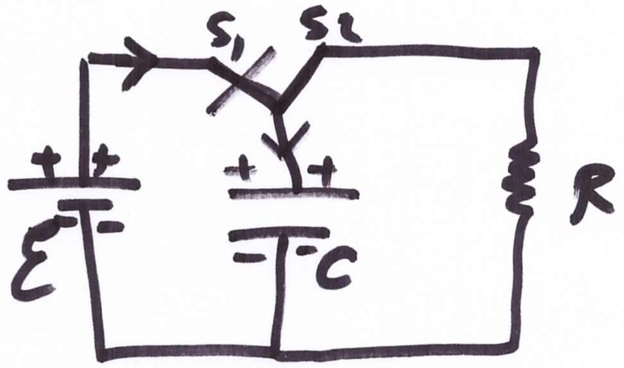
$$q = q_{max}\, e^{-t/RC}$$


I
$\frac{\mathcal{E}}{R} = I_{max}$
t

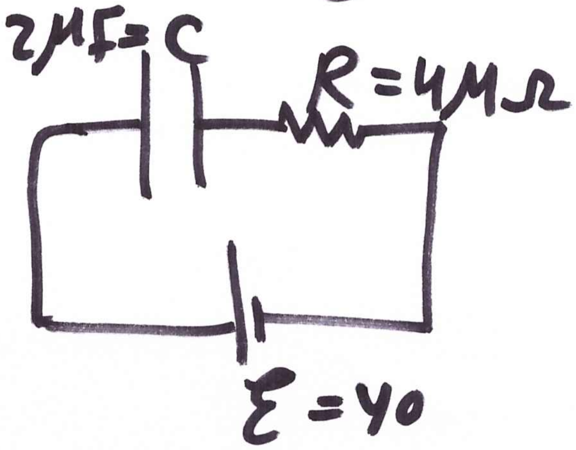

q
$C\mathcal{E} = q_{max}$
t


I
I_{max}
t


q
q_{max}
t


S_1
S_2
$\mathcal{E}$
C
R

Ex: RC circuit of $R = 4M\Omega$ and $C = 2\mu F$, R and C were Connected to a battery $\mathcal{E} = 40$ Volt as figure:



Find:

1) max. charge and current.
2) Time Constant (τ)
3) charge and Current at $t = 4$-sec.
4) Voltage of Capacitor and V. of resistor at $t = 4$ sec.
5) charge and current after 4-time Const.
6) time needed to reach to half of max. Current (or half of resistor voltage)
7) time needed to reach to 30% of max charge.
8) energy in capacitor after 4 sec.

$\boxed{C = 2\mu F \quad R = 4M\Omega \quad \mathcal{E} = 40\,Volt}$

1) $q_{max} = C\mathcal{E} = 2\mu * 40 = 80\mu C.$

$I_{max} = \frac{\mathcal{E}}{R} = \frac{40}{4*10^6} = 10\mu A.$

2) $\tau = RC = 4*10^6 * 2*10^{-6} = 8\text{-sec}.$

3) $q = q_{max}(1 - e^{-t/RC})$

$= 80*10^{-6}(1 - e^{-4/8})$

$= 31.5*10^{-6}\ C$

$I = I_{max}\ e^{-t/RC}$

$= 10*10^{-6} * e^{-4/8}$

$= 6.1*10^{-6} A$

4) $V_c = \frac{q}{C} = \frac{31.5\mu}{2\mu} = 15.75\ V.$

$V_R = IR = 6.1*10^{-6} * 4*10^6 = 24.4\ Volt$

$V_R = \mathcal{E} - V_c = 40 - 15.75 = 24.25\ Volt.$

5) $q = 80*10^{-6}[1 - e^{-4\cancel{\tau}/\cancel{\tau}}]$

$= 78.5*10^{-6} C.$

$I = 10*10^{-6}\ e^{-4\cancel{\tau}/\cancel{\tau}}$

$= 0.18\mu A.$

6) $I = I_{max}\, e^{-t/\tau}$

$\frac{1}{2}\cancel{I_{max}} = \cancel{I_{max}}\, e^{-t/\tau}$

$\frac{1}{2} = e^{-t/\tau} \Longrightarrow \ln 0.5 = \cancel{\ln}\, \cancel{e}^{-t/8}$

$\cancel{-}0.69 = \cancel{-}\frac{t}{8}$

$\boxed{t = 5.52\ sec}$

7) $q = q_{max}\left(1 - e^{-t/\tau}\right)$

$\frac{30}{100}\cancel{q_{max}} = \cancel{q_{max}}\left(1 - e^{-t/8}\right)$

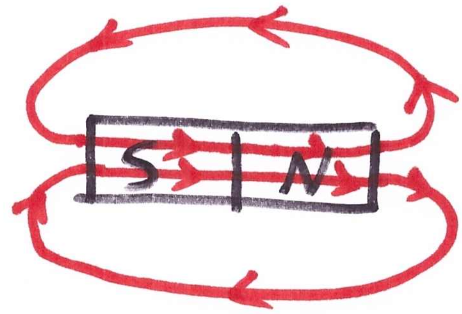
$0.3 = 1 - e^{-t/8}$

$\cancel{-}0.7 = \cancel{-}e^{-t/8}$ $(\ln)$

$\ln 0.7 = \cancel{\ln e}^{-t/8}$

$-0.36 = -\frac{t}{8}$

$\boxed{t = 2.88\ sec}$

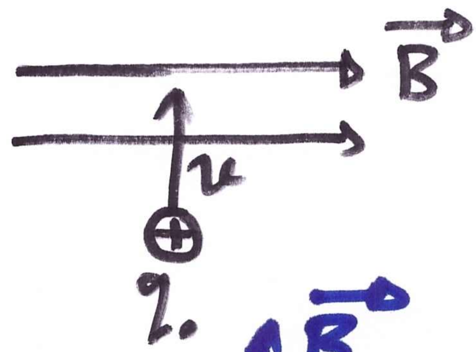


$\hat{k}$ ⊙ $\qquad$ $-\hat{j}$ ↓ $\qquad$ ↑ $\hat{j}$ $\qquad$ → $+\hat{i}$

$-\hat{k}$ ⊗ $\qquad$ ← $-\hat{i}$

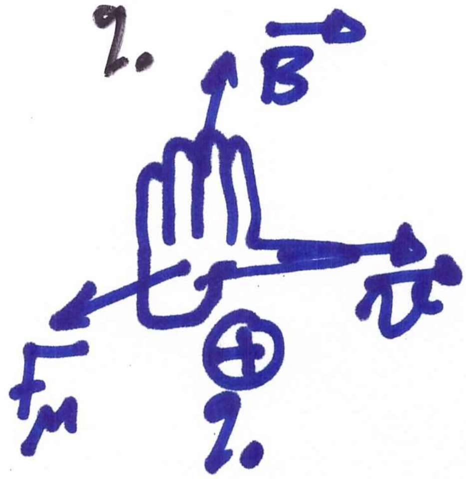
$$\vec{F}_M = q\,\vec{v} \times \vec{B}$$

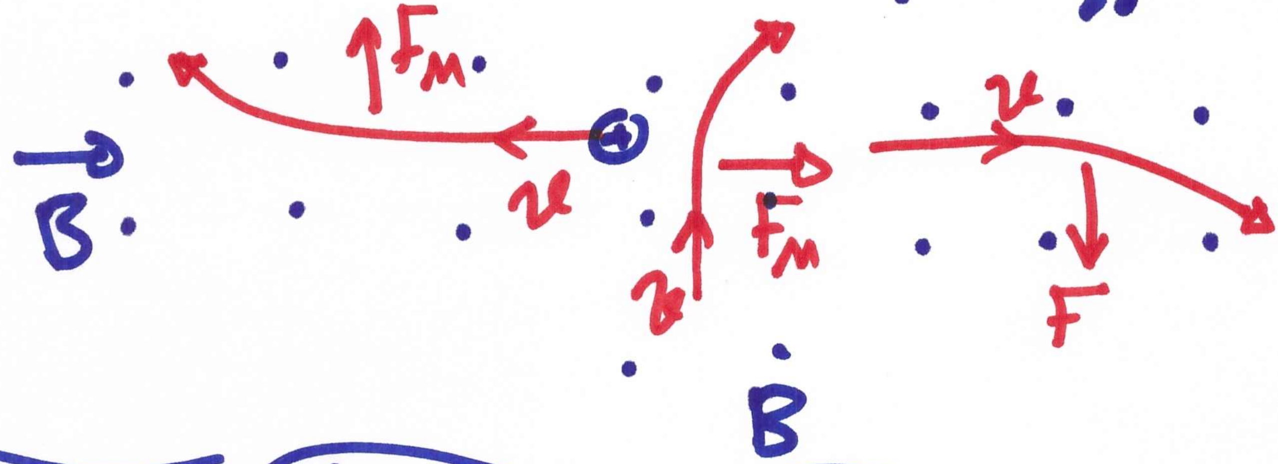
$$F = q_o\, v B \sin\theta_{vB}$$



$F = 0$ → $v = 0$ (rest)

→ $\theta = 0^\circ, 180^\circ$





$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \quad \| \quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta_{AB}$$

$$= (A B - A B_x)\hat{i} - (A B - A B)\hat{j} + (A B - A B)\hat{k}$$

Ex: If $\vec{v}$ is given by:

$$\vec{v} = 2\hat{i} - 3\hat{j} + \hat{k}$$

and $\vec{B} = \hat{i} + 2\hat{j} + 2\hat{k}$. $q_o = 2 \times 10^{-4}\,C$

Find: 1) Force as a vector.

2) magnitude of the force.

1)

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= (-6 - 2)\hat{i} - (4 - 1)\hat{j} + (4 + 3)\hat{k}$$

$$\vec{v} \times \vec{B} = -8\hat{i} - 3\hat{j} + 7\hat{k}$$

$$\vec{F}_M = q_o \vec{v} \times \vec{B} = 2 \times 10^{-4}[-8\hat{i} - 3\hat{j} + 7\hat{k}]$$

$$= (-16\hat{i} - 6\hat{j} + 14\hat{k}) \times 10^{-4} \text{ New.}$$

2) $|\vec{F}_M| = \sqrt{16^2 + 6^2 + 14^2} \times 10^{-4} = 22 \times 10^{-4}\,N.$

* If: find $\theta_{vB} \Rightarrow |\vec{F}| = q_o v B \sin\theta_{vB}$

$$|\vec{v}| = v = \sqrt{2^2+3^2+1^2} = \sqrt{14}$$

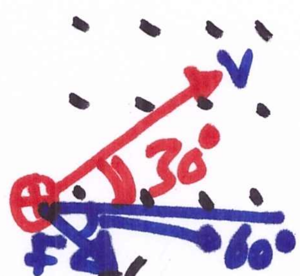
$$|\vec{B}| = B = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$$

$$\Rightarrow 22 * \cancel{10^{-4}} = 2 * \cancel{10^{-4}} * \sqrt{14} * 3 \sin\theta_{vB}$$

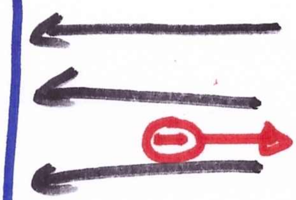
$$\sin\theta = 0.98 \Rightarrow \boxed{\theta = 78.5}$$

Ex: In the figures, Find $\vec{F}_M$:

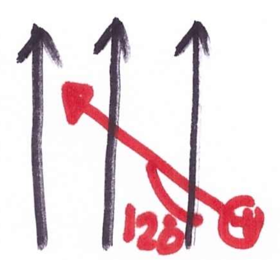
$|q| = 3\times10^{-6}C$
$v = 10m/s$
$B = 2$ Tesla



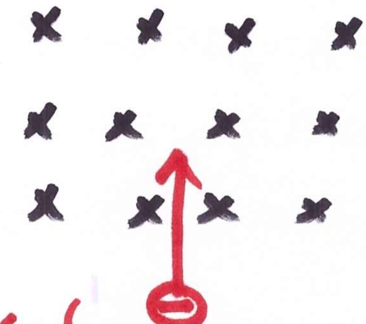
$F_M = 60\times10^{-6}\sin 90$
$= 60 * 10^{-6}N$



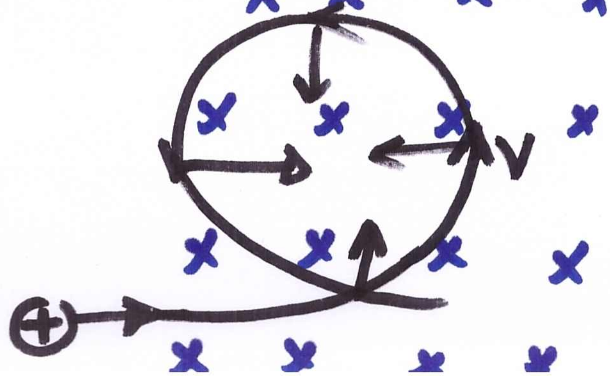
$F_M = 0$



$F_M = 60\times10^{-6}\sin 60$
$= 30\sqrt{3} * 10^{-6}N$
$(-\hat{k})$



$F_M = 3\times10^{-6}\times10\times2\sin 90$
$= 60 * 10^{-6}N$
$(+\hat{i})$



Circular motion.
F_m toward Center

$$F_c = \frac{mv^2}{r}$$

$$a_c = \frac{v^2}{r}$$

$\downarrow$
centripetal accel.
التسارع المركزي

$$q\cancel{v}B = \frac{mv^{\cancel{2}}}{r}$$

$$\boxed{r = \frac{mv}{qB}}$$

$$v = \frac{2\pi r}{T_p}$$

T_p : Period الزمن الدوري

$$T_p = \frac{2\pi r}{v} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

$f \equiv$ frequency $\rightarrow f = \frac{1}{T_p} = \frac{v}{2\pi r} = \frac{qB}{2\pi m}$
التردد

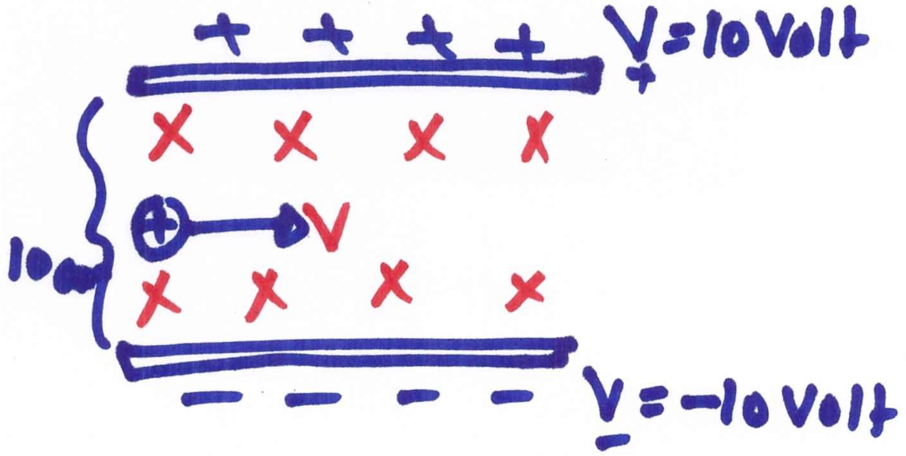
* $\omega \equiv$ angular freq. $\Rightarrow \omega = 2\pi f = \frac{v}{r} = \frac{qB}{m}$

Ex: If $q = 4\mu C$ entered a Uniform mag. field $B = 4T$ with const speed $v = 10\,m/s$ and if $m = 2\times10^{-5}\,kg$, Find:

1) magnetic force.
2) centripetal force

$$F_M = F_c = \begin{cases} qVB\sin\theta \checkmark \\ \frac{mv^2}{r} \times \end{cases}$$

3) " accel. $\Rightarrow a_c = \frac{F_c}{m}$
4) radius. $\Rightarrow r = \frac{mV}{qB}$
5) Period $T = \frac{2\pi r}{V} = \frac{2\pi m}{qB}$
6) freq. $\longrightarrow f = \frac{1}{T}$
7) ang. freq. (velocity) $\rightarrow \omega = 2\pi f = \frac{2\pi}{T}$



$B = 20\,T$

$q_0 = 4\times10^{-6}\,C$

$v = 20\,m/s$

① E

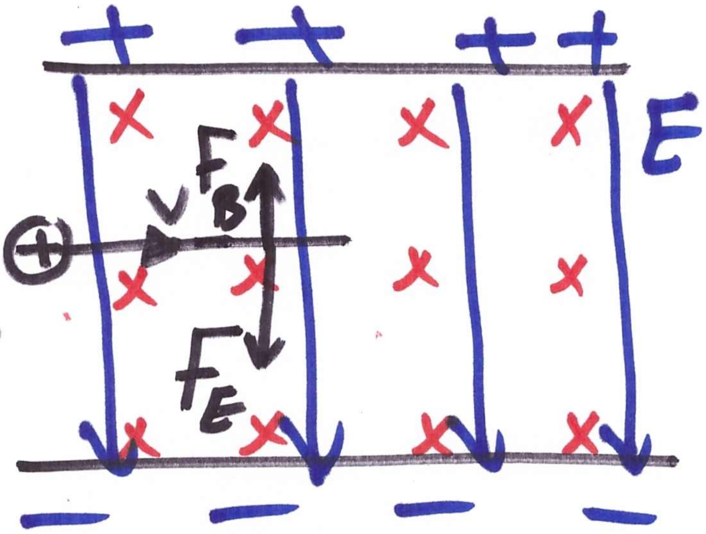
② F_E

③ $F_B = F_m$

④ Lorentz force

⑤ V to ensure q moves in a straight line

9.



① $\Delta V = Ed$

$(10 - -10) = E \; 10\times10^{-2}$

$E = \frac{20}{10\times10^{-2}} = 200 \; N/C$

② $F_E = qE = 4\times10^{-6} * 200 = 800*10^{-6} N. \; (-\hat{j})$

③ $F_B = qvB\sin\theta = 4\times10^{-6} \times 20 \times 20 \sin 90$

$= 1600*10^{-6} N \; (+\hat{j})$

④ $\vec{F}_{lorentz} = \vec{F}_E + \vec{F}_B$

$F_{lor} = F_B - F_E = 800*10^{-6} (+\hat{j})$

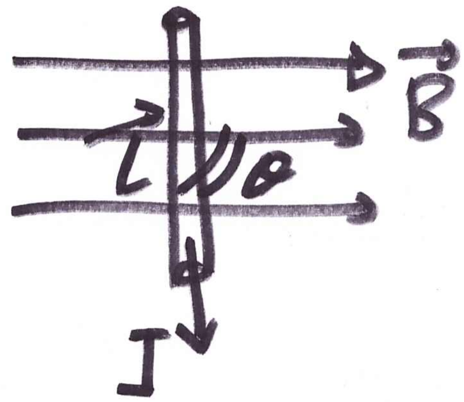
⑤ $F_E = F_B$

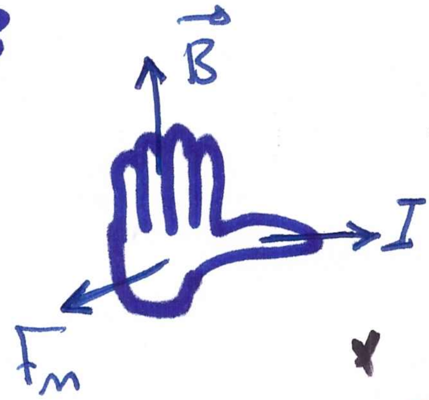
$\cancel{q}E = \cancel{q}vB$

$V = \frac{E}{B} \Rightarrow V = \frac{200}{20} = 10 m/s.$

$$\vec{F}_B = I\,\vec{l} \times \vec{B}$$

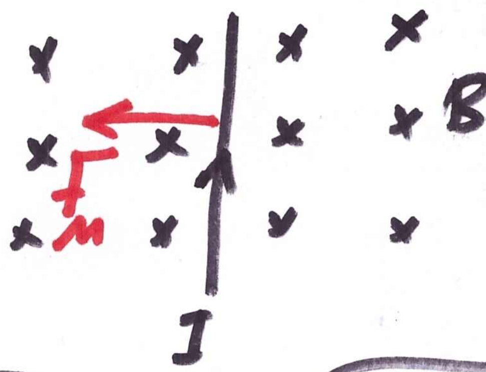
$$F_B = I\,l\,B\,\sin\theta_{IB}$$





$F = 0$

$\hookrightarrow \theta = 0°, 180°.$

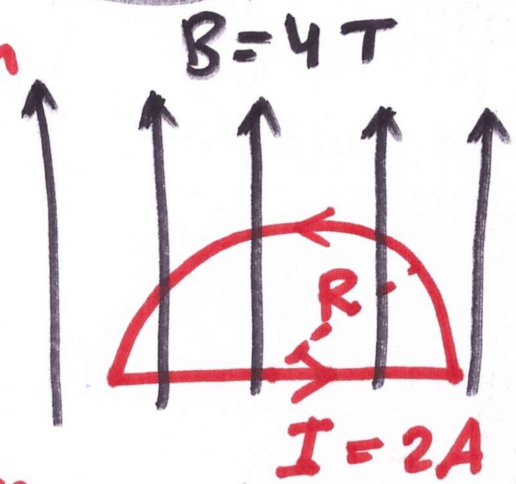


Ex: Find:

1) Net force on the current loop

2) Force on the stright wire

3) " " " Bented wire



① $F_{Net} = 0$

② $\vec{F} = I\,l\,B\sin\theta$

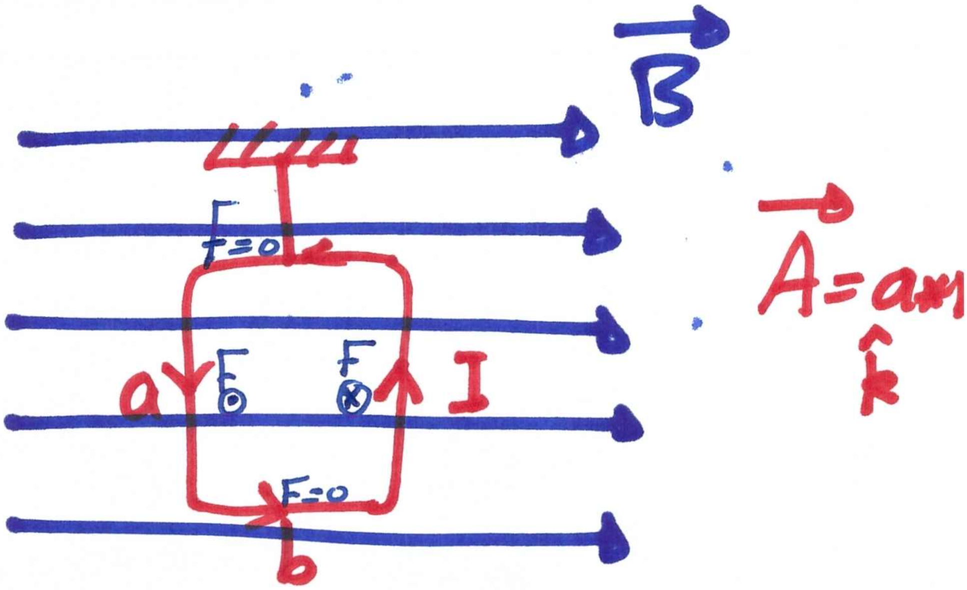
$= 2 * 20 \times 10^{-2} * 4 \sin 90$

$= 1.6$ New. $[2IRB]\,\hat{k}$

③ $F_m = 2IRB\,(-\hat{k})$

$= 1.6(-\hat{k})$ New.

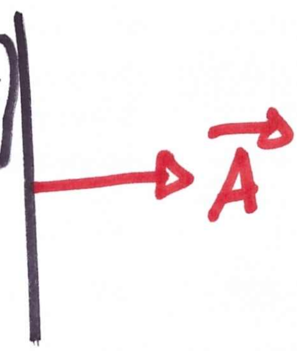
Torque: (τ)



$$\vec{\tau} = NI\vec{A} \times \vec{B}$$

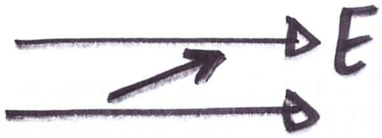
$$\tau = NIAB \sin\theta_{AB}$$

* $\vec{\mu} = NI\vec{A}$ [dipole magnetic moment]

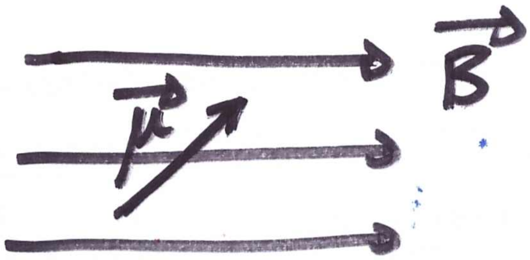


$$\Rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\tau = \mu B \sin\theta_{\mu B}$$



$U = -\vec{P}\cdot\vec{E}$



U: Potential energy of magnetic dipole moment in $\vec{B}$

$U = -\vec{\mu}\cdot\vec{B}$

$= -\mu B \cos\theta_{\mu B}$

$U_{max} = \mu B \quad (\theta = 180°)$

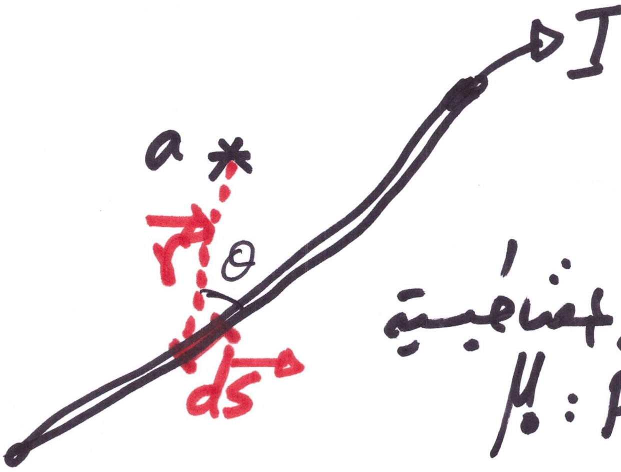
$U_{min} = -\mu B \quad (\theta = 0)$

انتهى CH:29

CH:30 Sources of mag. Field

1 Biot - Savart law :-

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$



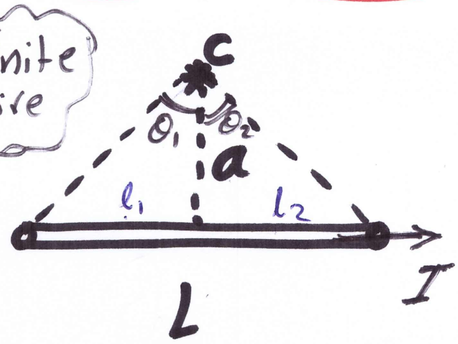
$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}}$$

النفاذية المغناطيسية

μ_0 : Permeability

$\mu_0 = 4\pi \times 10^{-7}\ T.m/A$

(a) Finite wire



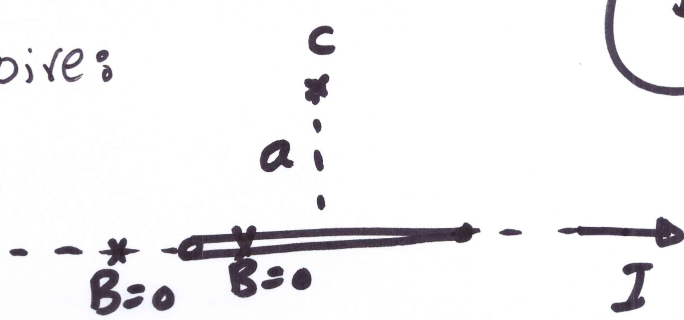
$$B_c = \frac{\mu_0 I}{4\pi a}[\sin\theta_1 - \sin\theta_2]$$

$$\sin\theta_1 = \frac{l_1}{\sqrt{l_1^2 + a^2}}$$

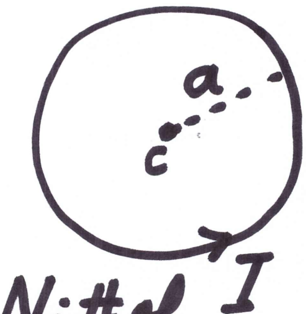
$$\sin\theta_2 = \frac{l_2}{\sqrt{l_2^2 + a^2}}$$

(b) Infinite wire:

$$B_c = \frac{\mu_0 I}{2\pi a}$$



(c) circular wire



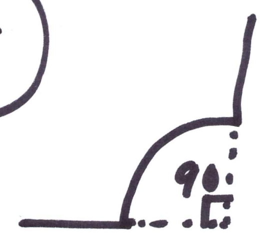
N: # of turns

$$B = \frac{\mu_0 N I}{2a} = \frac{\mu_0 I \theta}{4\pi a}$$

$$N = \frac{\theta}{2\pi}$$

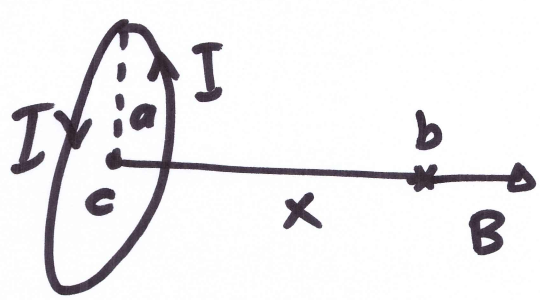
$N = \frac{1}{2}$

$N = \frac{1}{6}$



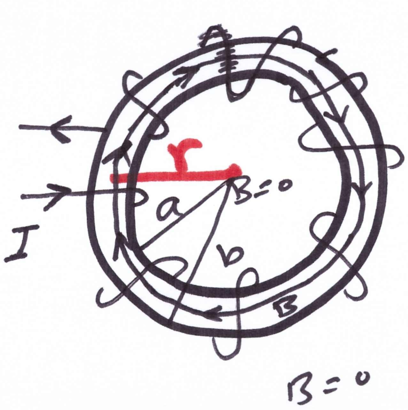
$N = \frac{1}{4}$

(d)



$$B_b = \frac{\mu_0 I a^2 N}{2(a^2+x^2)^{3/2}}$$

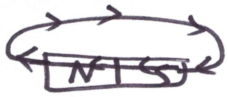
(e) Toroid:

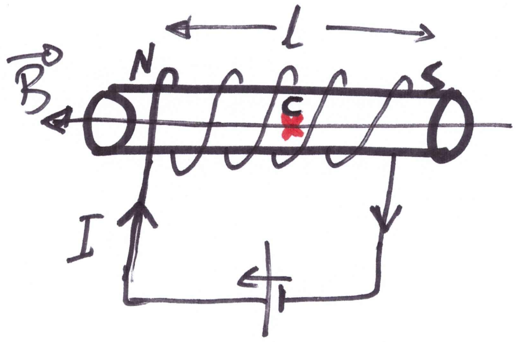


$$B = \frac{\mu_0 N I}{2\pi r}$$

$a < r < b$

(f) Solenoid



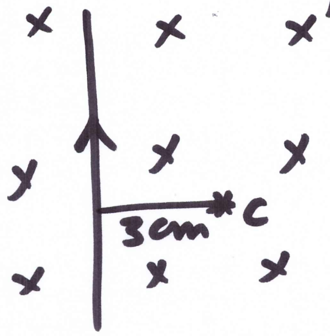


$$B_c = \frac{\mu_0 N I}{L} = \mu_0 n I$$

$\frac{N}{L} = n$: # of turns per unit length

Find the net $\vec{B}$ at c :-

Ex: $B = 4*10^{-5}$ T



$I = 6A$

$B_1 \equiv \otimes \; (-\hat{k})$

$B_2 \equiv \otimes \; (-\hat{k})$
straight

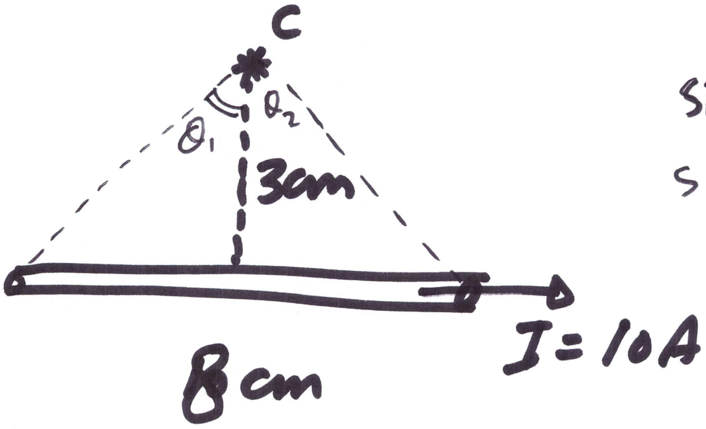
١) الاتجاهات
٢) الحسابات
٣) المحصلة

$B_1 = 4*10^{-5}$ T

$$B_2 = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \times 10^{-7} * 6}{2\pi\, 3\times 10^{-2}} = 4*10^{-5} \text{ T}.$$

$$B_c = B_1 + B_2 = 8*10^{-5} \text{ T} \otimes (-\hat{k}).$$

Ex:



$\sin\theta_1 = \frac{4\,cm}{5\,cm} = 0.8$

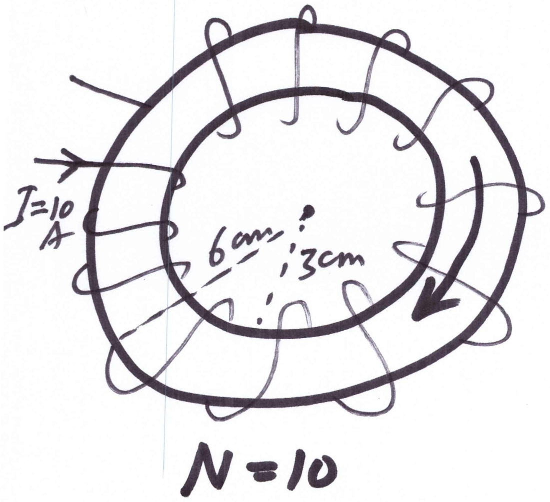
$\sin\theta_2 = \frac{-4\,cm}{5\,cm} = -0.8$

$$B_c = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

$$= \frac{\cancel{4\pi}\times10^{-7} * 10}{\cancel{4\pi}\, 3*10^{-2}}(0.8 - (-0.8))$$

$$= \frac{16}{3}*10^{-5}\ T$$

Ex: Find B at 5cm from the center:

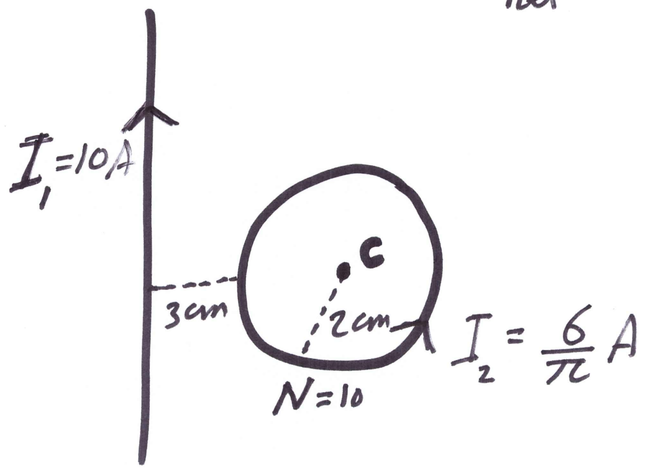


$$B = \frac{\mu_0 N I}{2\pi r}$$

$$= \frac{\cancel{4}\cancel{\pi}\times10^{-7} * 10 * \cancel{10}}{\cancel{2}\cancel{\pi}\ \cancel{5}\times10^{-2}}$$

$$= 40 * 10^{-7}\ T$$

Ex: Find B_{net} at $\underline{c}$:-



$*^{C}$ $B_1 \otimes$

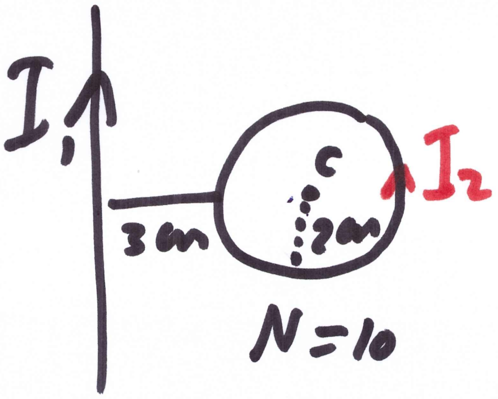
$B_2 \odot$

$$B_1 = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \times 10^{-7} * 10}{2\pi\, 5*10^{-2}} = 4*10^{-5}\, T$$

$$B_2 = \frac{\mu_0 I N}{2a} = \frac{4\pi * 10^{-7} * 10 * 6}{2*2\times 10^{-2}\pi} = 60*10^{-5}\, T$$

$$B_{net} = B_2 - B_1 = 56*10^{-5}\, T \odot (+\hat{k})$$

Ex: In the previous example, Find $\underline{I_2}$ if B at $\underline{c}$ is $1*10^{-5}\, T\ (-\hat{k})$



$*^{C}$

$B_1 \otimes = 4*10^{-5}\, T$

$B_c \otimes = 1*10^{-5}\, T$

$B_2 \odot = 3*10^{-5}\, T$

$\boxed{\text{if } B_c = 0}$

$B_1 = B_2$

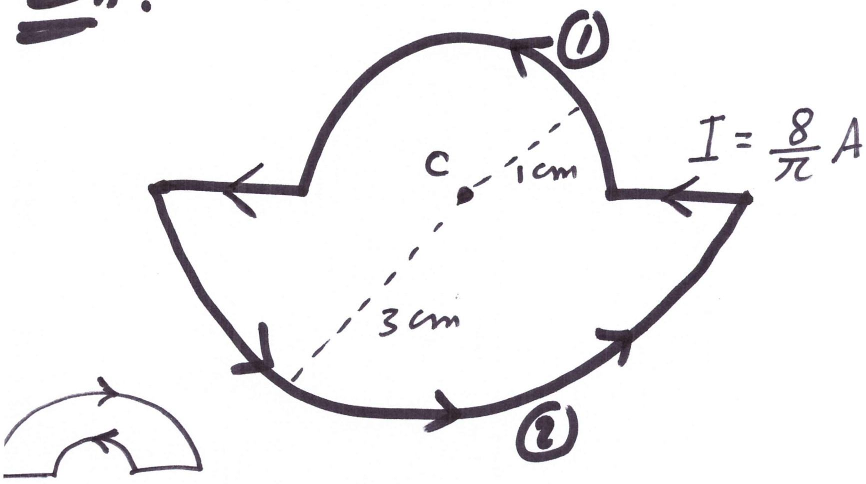
$$\frac{\mu_0 I_1}{2\pi a_1} = \frac{\mu_0 I_2 N}{2 a_2}$$

$$\frac{10}{\pi\, 5\times 10^{-2}} = \frac{I_2 * 10}{2cm}$$

$$I_2 = \frac{0.4}{\pi} A$$

$$B_2 = \frac{\mu_0 I N}{2*a} \Rightarrow 3*10^{-5} = \frac{4\pi\times 10^{-7} I_2\, 10}{2*2*10^{-2}} \Rightarrow I_2 = \frac{0.3}{\pi} A$$

Ex:



c × B_1 ⊙

B_2 ⊙

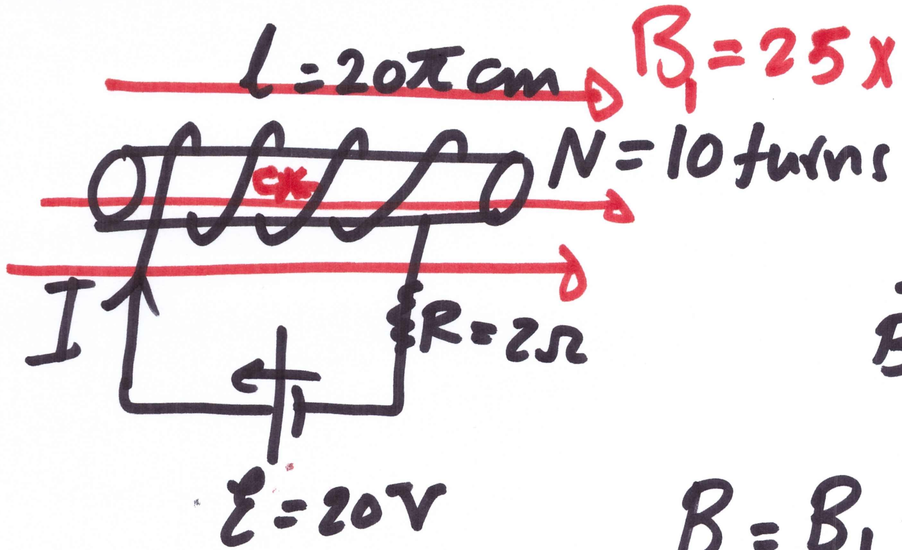
$$B_1 = \frac{\mu_0 N I}{2a} = \frac{4\pi \times 10^{-7} * 1 * 8}{2 \times 1 \times 10^{-2} * 2 * \pi} = 8 * 10^{-5} T$$

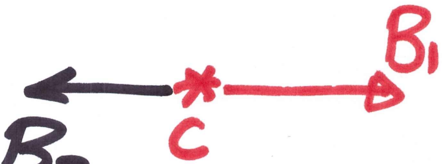
$$B_2 = \frac{4\pi \times 10^{-7} * 1 \times 8}{2 \times 3 \times 10^{-2} \; 2 * \pi} = \frac{8}{3} \times 10^{-5} T$$

$$B_c = B_1 + B_2 = \frac{32}{3} * 10^{-5} T \odot (+\hat{k}).$$

Ex:

$B_1 = 25 \times 10^{-5} T$





$$I = \frac{\varepsilon}{R} = 10 A$$

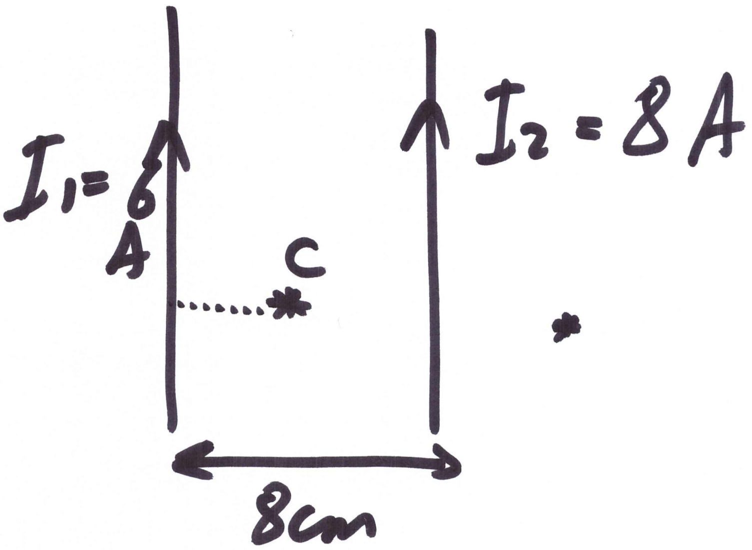
$$B_c = B_1 - B_2$$

$$= 25 \times 10^{-5} - \frac{4\pi \times 10^{-7} \times 10 \times 10}{20\pi \times 10^{-2}}$$

$$= 25 \times 10^{-5} - 20 * 10^{-5}$$

$$= 5 \times 10^{-5} T \; (\hat{i})$$

Ex :



C $*$ $B_1 \otimes$

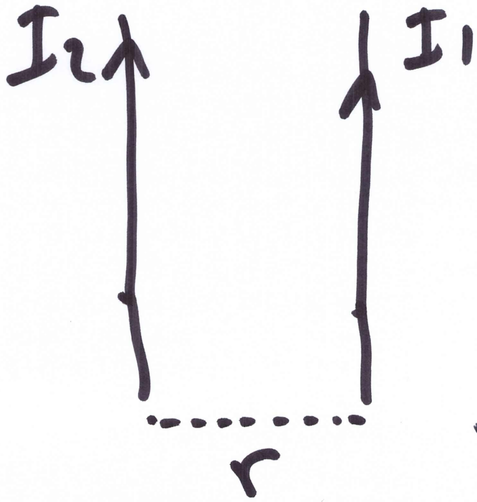
$B_2 \odot$

$$B_1 = \frac{4\pi \times 10^{-7} \times 6}{2\pi \, 4\times 10^{-2}} = 3\times 10^{-5}\,T$$

$$B_2 = \frac{4\pi \times 10^{-7} \times 8}{2\pi \times 4\times 10^{-2}} = 4\times 10^{-5}\,T$$

$$B_c = B_2 - B_1 = 1\times 10^{-5}\,T \odot \ (+\hat{k}).$$

نقطة التعادل المغناطيسي :



$$B_1 = B_2$$

متعاكسان

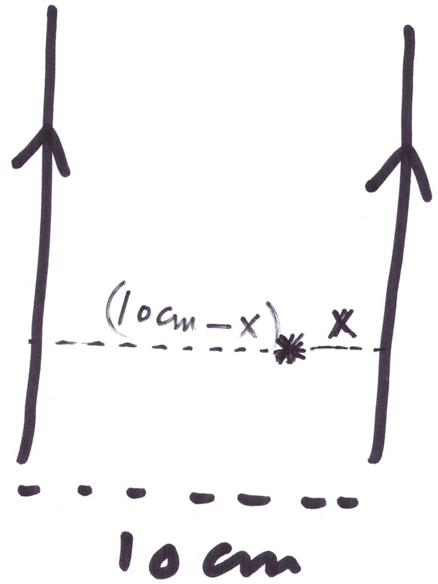
↑ ↑ ⇐ بينهما أقرب للأصغر

↓ ↑ ⇐ خارجهما أقرب للأصغر

Ex: Find the Point of equilibrium (where 8 $B_{net} = 0$)

$I_1 = 4A$

$I_2 = 6A$



$$B_1 = B_2$$

$$\frac{\cancel{\mu_0} I_1}{\cancel{2\pi} a_1} = \frac{\cancel{\mu_0} I_2}{\cancel{2\pi} a_2}$$

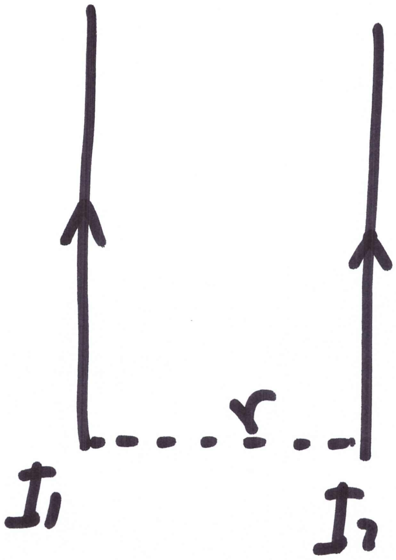
$$\frac{4}{x} = \frac{6}{10\times10^{-2} - x}$$

$$\Rightarrow 40\times10^{-2} - 4x = 6x$$

$$10x = 40\times10^{-2}$$

$$x = 4\times10^{-2}\ m = 4\ cm.$$

Force between the two wires:-



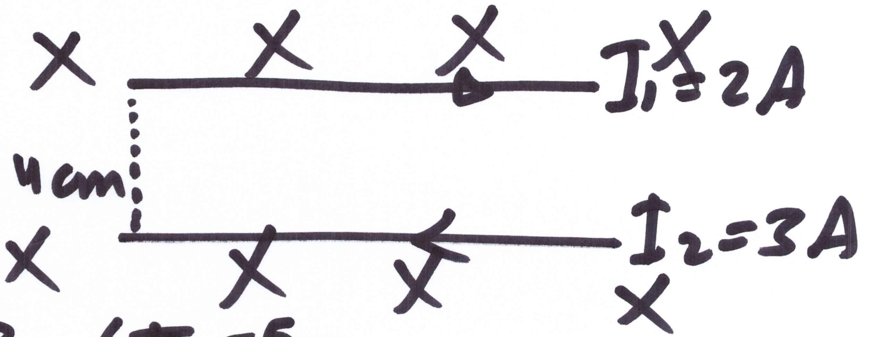
* $\uparrow\uparrow \Rightarrow$ تجاذب

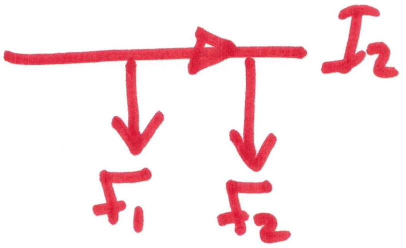
$\uparrow\downarrow \Rightarrow$ تنافر

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

$$\boxed{\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

Ex: what is the net Forces on I_2:





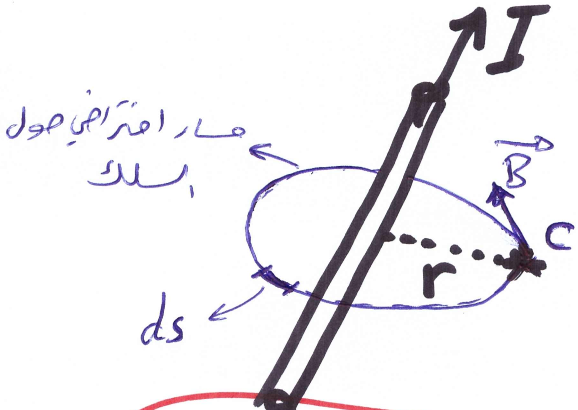
$B = 6*10^{-5}\,T$

$$\frac{F_1}{L} = \frac{\cancel{4\pi}\times 10^{-7} * 2*3}{2\cancel{\pi}\, 4\times 10^{-2}} = 3*10^{-5}\,N/m.$$

$$\boxed{\frac{F_{net}}{L} = \frac{F_1+F_2}{L} = 21\times 10^{-5}\,\frac{N}{m}\,(-\hat{j})}$$

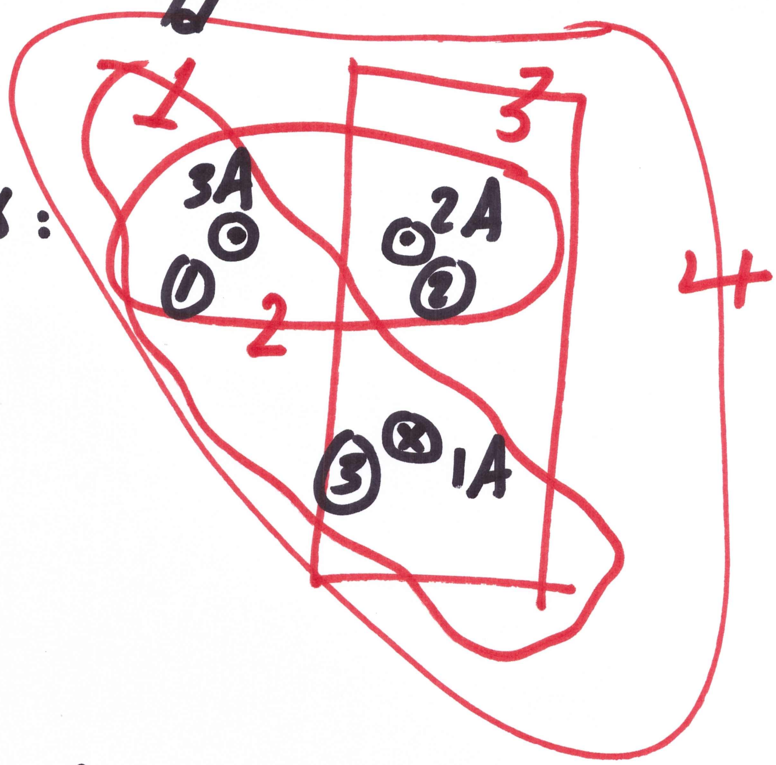
$$\frac{F_2}{l} = I_2 B \sin\theta = 3*6*10^{-5}\sin 90 = 18*10^{-5}\,N/m$$

§3 Ampere's law :-



$$\boxed{\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{ins}}$$

Ex :



Find the line integral of $\vec{B}\cdot d\vec{s}$ $\left(\oint \vec{B}\cdot d\vec{s}\right)$ for each line.

1) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{ins} = 4\pi \times 10^{-7} \times (3-1)$

$= 8\pi \times 10^{-7}$ T.m

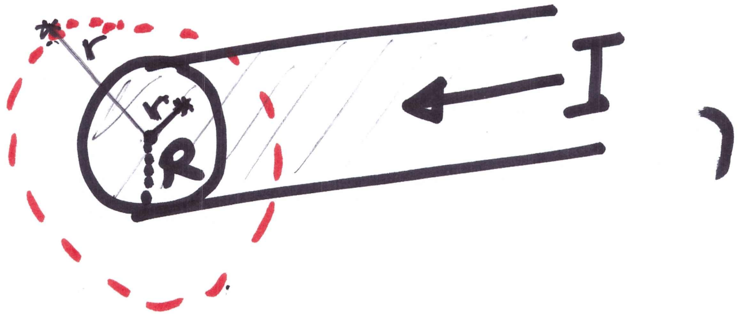
2) $\oint \vec{B}\cdot d\vec{s} = \mu_0(3+2) = 5\mu_0$ T.m

3) $\oint \vec{B}\cdot d\vec{s} = \mu_0(2-1) = \mu_0$ T.m.

4) $\oint \vec{B}\cdot d\vec{s} = \mu_0(3+2-1) = 4\mu_0$ T.m.

Ex: Find $\vec{B}$ at 1) $r \geq R$

2) $r < R$



1) $r \geq R$

$$\oint B \cdot d\vec{s} = \mu_0 I_{ins}$$

$\int ds = S_{total}$

$= 2\pi r$

$$B(2\pi r) = \mu_0 I$$

$$\boxed{B_{out} = \frac{\mu_0 I}{2\pi r}}$$

2) $\underline{r < R}$

$$B * S = \mu_0 I_{ins}$$

$$B * (2\pi r) = \mu_0 I'$$

$$\frac{I}{I'} = \frac{\cancel{\pi} R^2}{\cancel{\pi} r^2}$$

$$I' = I \frac{r^2}{R^2}$$

$$B = \frac{\mu_0 I r^{\cancel{2}}}{2\pi \cancel{r} R^2}$$

$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

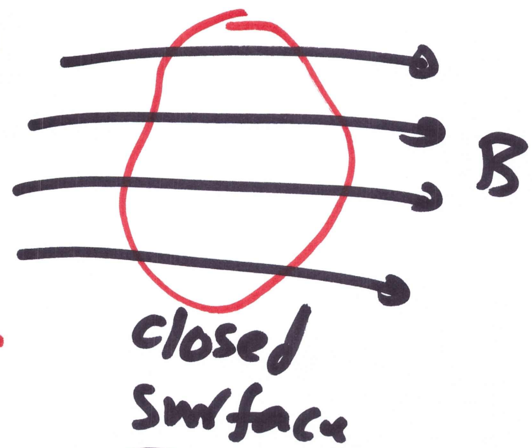
§5 : Gauss's law of magnetism

Φ_B : magnetic Flux التدفق المغناطيسي

$$\boxed{\Phi_B = \int \vec{B} \cdot d\vec{A} = \vec{B} \cdot \vec{A} = BA \cos\theta_{BA}}$$

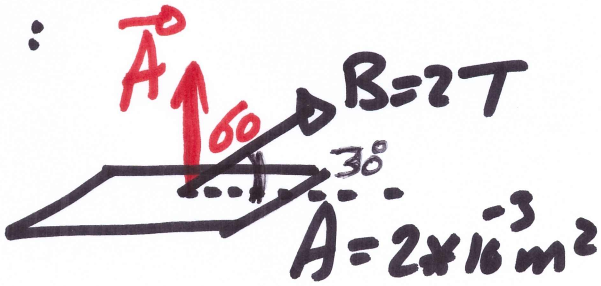
* The net magnetic flux (Φ_B) through any closed surface is always Zero.

$$\boxed{\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0}$$



$\Phi_B = 0$

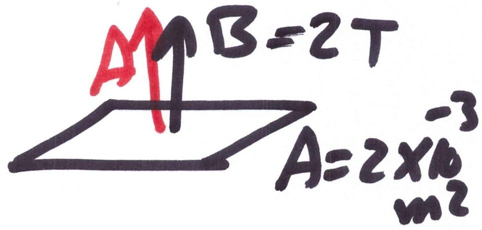
Ex :



$\Phi = BA\cos\theta$

$= 2 \times 2 \times 10^{-3} \cos 60$

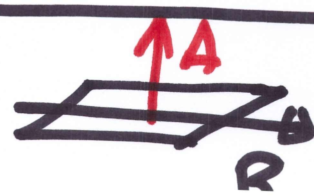
$= 2 \times 10^{-3}$ Weber



$\Phi = 2 \times 2 \times 10^{-3} \cos 0$

$\Phi_{max} = 4 \times 10^{-3}$ Wb.

$\Phi = 0$



Ex : If $\vec{B} = 4\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{A} = 4\hat{i} + 2\hat{j}$

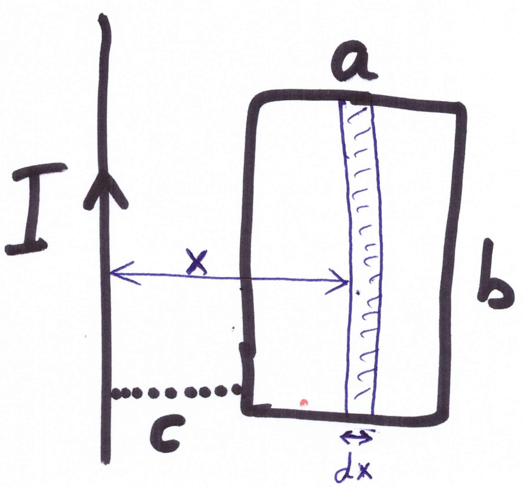
Find Flux (magnetic) and θ_{BA}

$$\Phi = \vec{B} \cdot \vec{A} = 16 + 6 = 22 \text{ Wb}.$$

$$\Phi = BA\cos\theta$$

$$22 = \sqrt{4^2+3^2+5^2} * \sqrt{4^2+2^2} \cos\theta_{BA}$$

Ex:



$x: c \longrightarrow c+a$

$dA = b * dx$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi x}$$

$$\Phi = \int \frac{\mu_0 I}{2\pi x} \cdot b\,dx$$

$$= \frac{\mu_0 I}{2\pi} b \int \frac{dx}{x}$$

$$= \frac{\mu_0 I b}{2\pi} \ln x \Big]_c^{c+a}$$

CH:30 انتهى

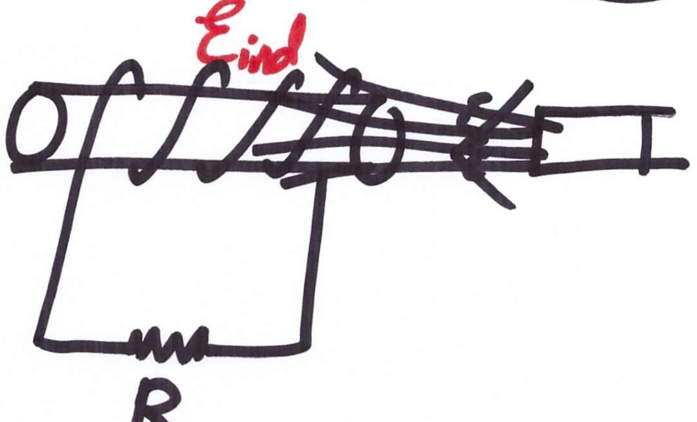
$$\Phi = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{c+a}{c}\right).$$

CH: 31 Faraday's law

$$\phi_B = \int \vec{B}\cdot d\vec{A} = \vec{B}\cdot\vec{A} = BA\cos\theta_{BA}$$

$$\Delta\phi = \begin{cases} \Delta B\, A\cos\theta \longrightarrow (\Delta B = B_2 - B_1) \\ B\,\Delta A\cos\theta \longrightarrow (\Delta A = A_2 - A_1) \\ B\,A\,\Delta\cos\theta \longrightarrow (\Delta\cos\theta = \cos\theta_2 - \cos\theta_1) \\ \phi_2 - \phi_1 = B_2A_2\cos\theta_2 - B_1A_1\cos\theta_1 \end{cases}$$

$$d\phi = d(BA\cos\theta) = A\cos\theta\, dB$$



$$\mathcal{E}_{ind} = -N\frac{d\phi}{dt}$$

$$\mathcal{E}_{ind} = -N\frac{\Delta\phi}{\Delta t}$$

$$I_{ind} = \frac{|\mathcal{E}_{ind}|}{R}$$

*magnitude of $\mathcal{E}_{ind}$ is $|\mathcal{E}_{ind}|$

Ex: A plane of dimentions 10cm x 6cm, and a Uniform mag. field $B = 4T$ directed out of page Perp. to the plane. Find the

1) electromotive force (induced).
2) induced current ($R = 2\Omega$).

if the B is dropped to zero through 0.2 sec.

$B = 4T \downarrow 0T$, $A = 60 \times 10^{-4}$ $\theta = 0$, $\Delta t = 0.2 sec.$

1) $\Delta\phi = \Delta B \, A \cos\theta$

$= (0 - 4)\, 60 * 10^{-4} \cos 0$

$= -240 \times 10^{-4}$ Wb.

$$\mathcal{E}_{ind} = -N \frac{\Delta\phi}{\Delta t} = -1 \times \frac{-240 \times 10^{-4}}{0.2} = +120 * 10^{-3} \text{ Wb/s}$$

2) $$I_{ind} = \frac{|\mathcal{E}|}{R} = \frac{120 * 10^{-3}}{2} = 60 * 10^{-3} A.$$

Ex: If $B = B_{max}\, e^{-\alpha t}$, Find the induced $\mathcal{E}$ through a surface of Area **A** parallel to the field. ($\theta = 0$)

Sol

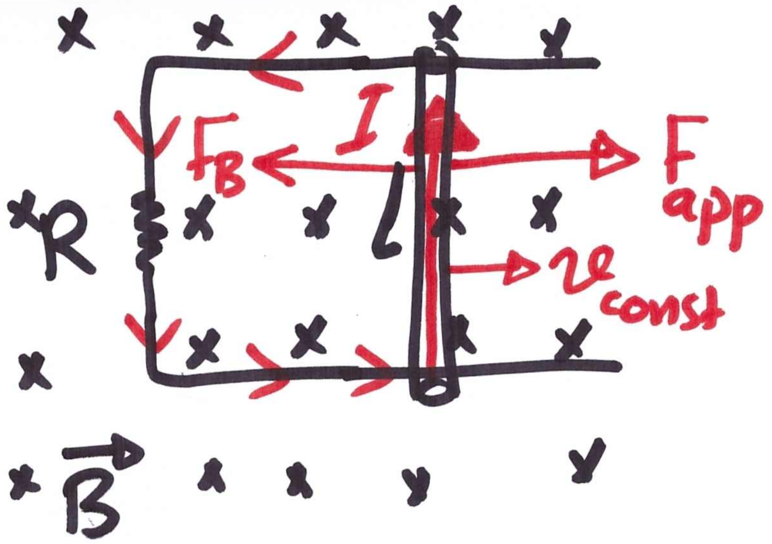
$$\phi = BA\cos\theta$$
$$= A B_{max}\, e^{-\alpha t}$$

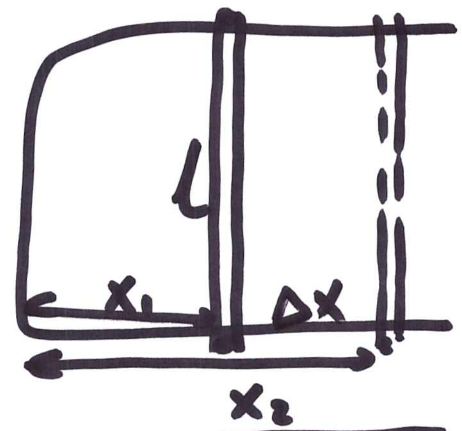
$$\frac{d\phi}{dt} = AB_{max}\left(-\alpha e^{-\alpha t}\right)$$
$$= \left(-\alpha A B_{max}\, e^{-\alpha t}\right)$$

$$\underset{\substack{ind \\ (ins)}}{\mathcal{E}} = -N\frac{d\phi}{dt} = \alpha A B_{max}\, e^{-\alpha t} .$$

$$* \; \underset{\substack{av \\ \text{from } t=0 \to 2}}{\mathcal{E}} = -\frac{\Delta\phi}{\Delta t} = \frac{\phi_2 - \phi_1}{t_2 - t_1}$$

$$\phi_1 = A B_1$$
$$\phi_2 = A B_2$$





$$\boxed{\Delta A = l * \Delta x}$$

$$\Delta \Phi = B \Delta A \cos\theta$$

$$\boxed{\begin{array}{l} \mathcal{E}_{in} = -\dfrac{\Delta \Phi}{\Delta t} \\ \mathcal{E}_{ind} = -l v B \end{array}}$$

$$v = \frac{\Delta x}{\Delta t}$$

$$\frac{|\mathcal{E}|}{R} = I$$

$$F_{app} = F_B = I l B \sin_{lB}$$

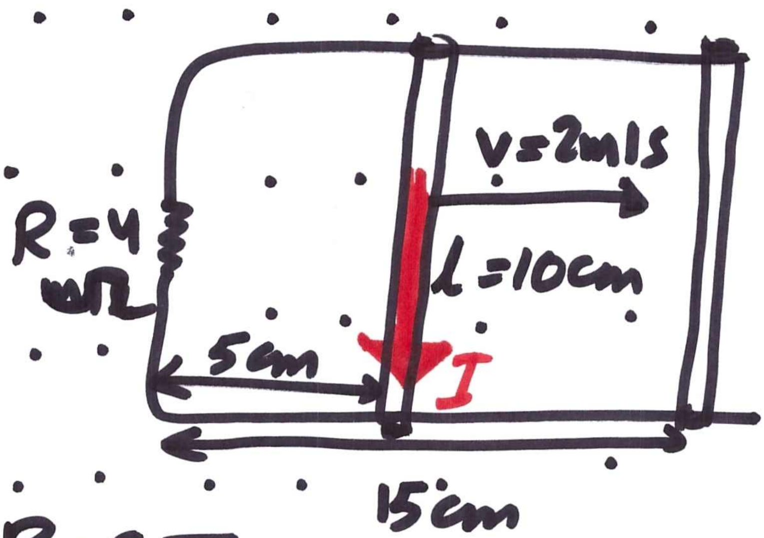
$$\text{Power} = F_{app} v = I l B v$$

$$\boxed{\text{Power} = \frac{B^2 l^2 v^2}{R} = \mathcal{E}^2 / R}$$

If $V_{ini} \neq 0$ (V is not constant)

$$\Rightarrow v = v_{initial}\, e^{-t/\tau} \qquad \left(\tau = \frac{mR}{B^2 l^2} \right)$$

Ex In the figure, Find:



B = 2T

1) ΔX
2) Δt
3) ΔA
4) $\Delta \phi$
5) $\mathcal{E}$
6) I
7) F_{app} , F_B
8) Power.

Sol

1) $\Delta X = 15 - 5\,cm$
$= 10 \times 10^{-2}\,m$

2) $V = \frac{\Delta X}{\Delta t}$
$\Rightarrow \Delta t = \frac{10 \times 10^{-2}}{2}$
$= 5 * 10^{-2}\,sec.$

3) $\Delta A = l * \Delta X$
$= 10 \times 10^{-2} * 10 \times 10^{-2}$
$= 100 * 10^{-4}\,m^2.$

4) $\Delta \phi = B\,\Delta A \cos\theta = 2 * 1 \times 10^{-2} \cos 0 = 2 \times 10^{-2}\,Wb$

5) $\mathcal{E}_{ind} = -\frac{\Delta \phi}{\Delta t} = -\frac{2 \times 10^{-2}}{5 \times 10^{-2}} = -0.4$ Volt.

6) $I_{in} = \frac{|\mathcal{E}|}{R} = \frac{0.4}{4} = 0.1\,A\,[\ -\hat{j}\ ]$

F_{app} $+i$

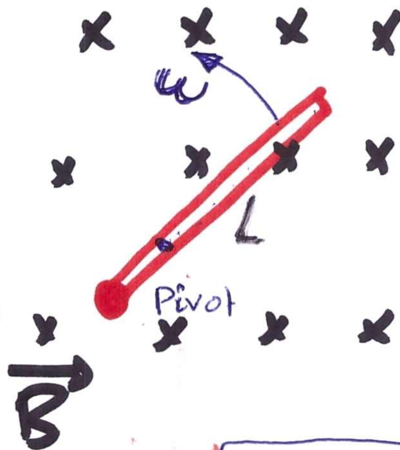
7) $F_{app} = F_B = I l B = 0.1 * 10 \times 10^{-1} * 2 = 2 \times 10^{-1}\,N$

$-i$ F_B

8) Power $= \mathcal{E}^2/R = 0.16/4 = 4 * 10^{-2}$ watt.

$$\boxed{\mathcal{E} = \frac{1}{2} B \omega l^2}$$

$$I_{in} = \frac{|\mathcal{E}|}{R} \qquad T = \frac{2\pi}{\omega}$$

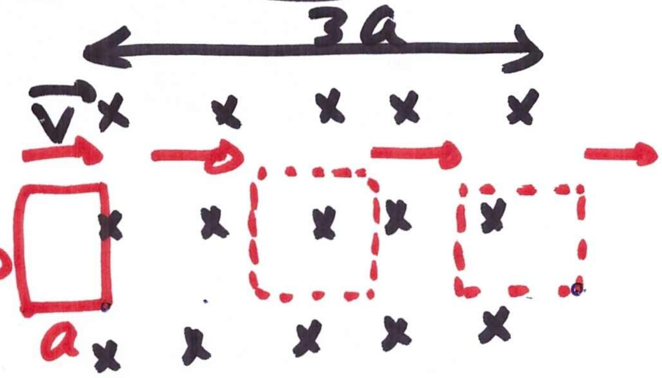


$$\boxed{\omega = \frac{v}{r}}$$

ω : angular Velocity (rad/sec)

T : Period

Ex 31.6

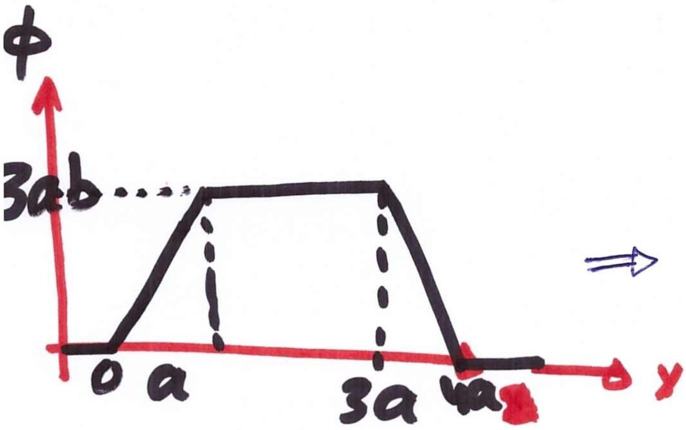


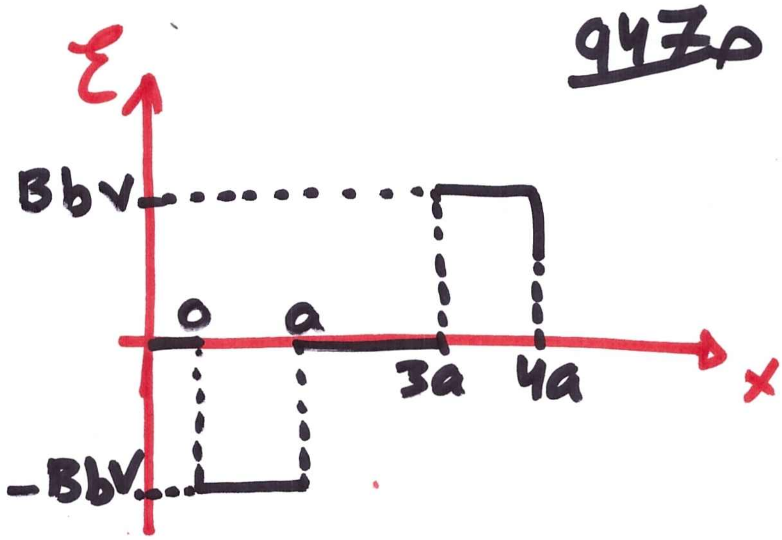
$$F = I l \times B \qquad I = \frac{|\mathcal{E}|}{R}$$

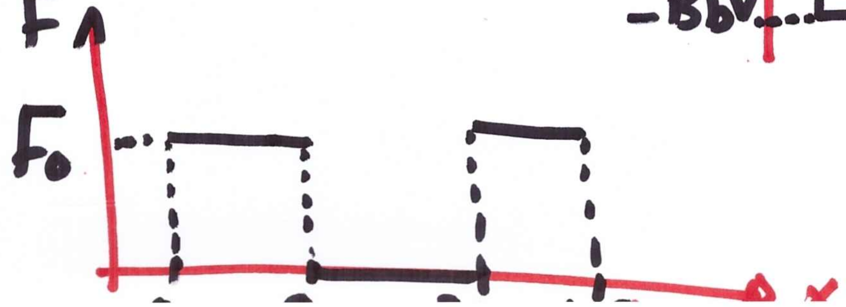
$$F_0 = \frac{v l^2 B^2}{R}$$

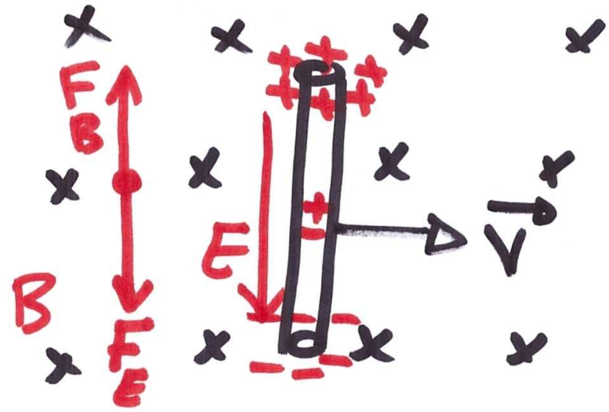
B

947o









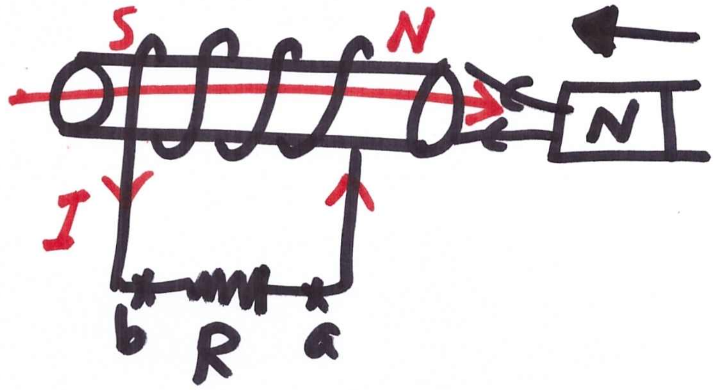
$F_B = q v B \sin\theta$

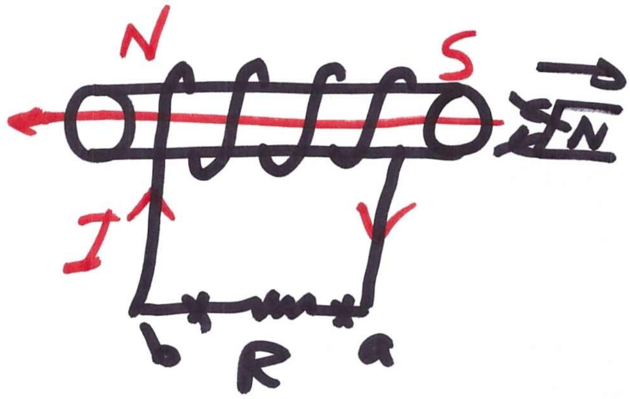
$F_E = F_B$

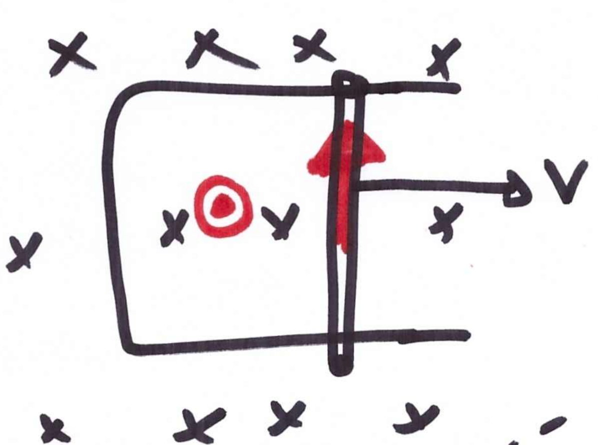
$qE = qvB$

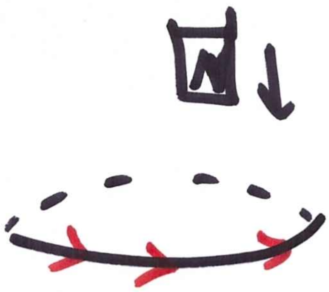
$v = \frac{E}{B}$

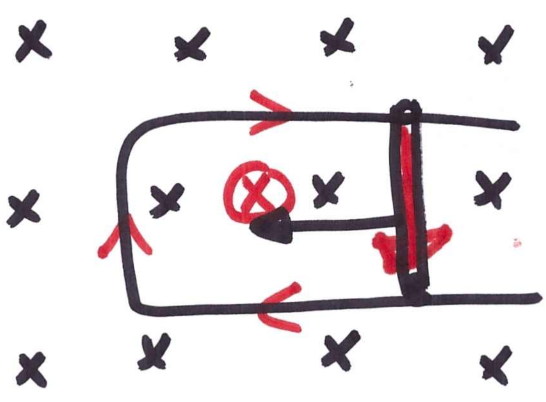
Lenz law :-



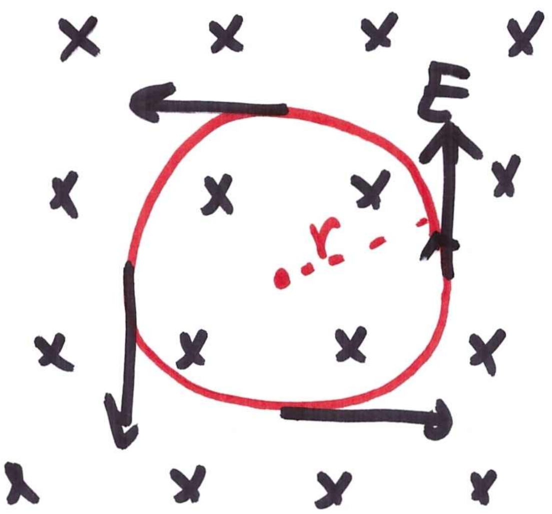








§4: Induced emf and Electric fields



B: depent on time

$B = B(t)$

$$\frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt} \qquad A \begin{cases} A = \pi r^2 \\ a \times b \end{cases}$$

$$\mathcal{E} = -N\frac{d\phi}{dt} = -NA\frac{dB}{dt}$$

$$\cancel{q}\mathcal{E} = \cancel{q}E(2\pi r) \qquad \Delta V = E \cdot l$$

$$E = \frac{\mathcal{E}}{2\pi r} = -\frac{1}{2\cancel{\pi}\cancel{r}} N\cancel{\pi}\cancel{r}^2\frac{dB}{dt} \qquad \frac{}{l}$$

$$E_{in} = -\frac{N}{2} r \frac{dB}{dt}$$

$$\oint E \cdot d\vec{s} = -N\frac{d\phi}{dt}$$

$$E \cdot \Delta L = \mathcal{E}$$

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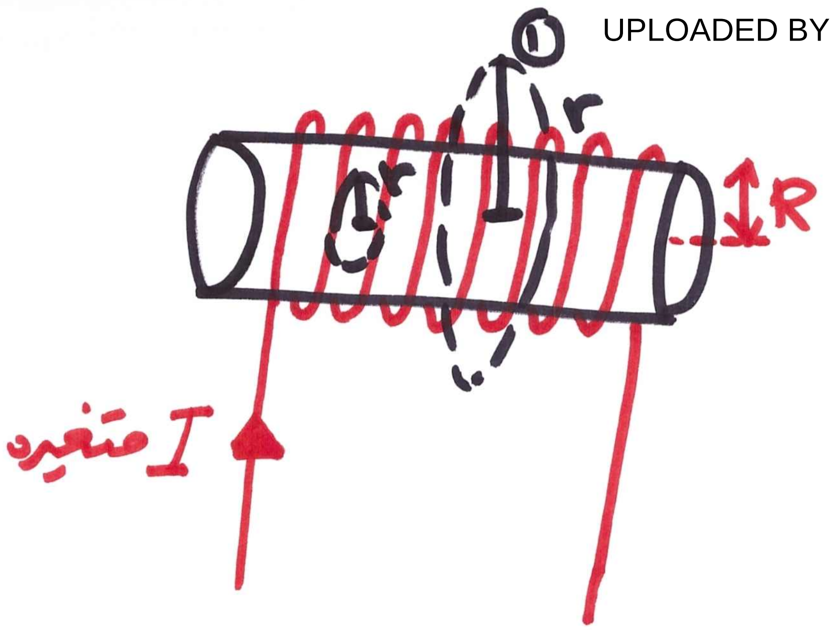

$n = \frac{N}{L}$

$$I = I_{max} \cos\omega t$$

1) r > R

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -\frac{d}{dt}(\mu_0 n I \pi R^2)$$

$$= -\mu_0 n \pi R^2 \frac{dI}{dt}$$

$$= -\mu_0 n \pi R^2 (-I_{max}\, \omega \sin\omega t)$$

$$\mathcal{E} = \mu_0 n \pi \omega R^2 I_{max} \sin\omega t .$$

$$E(\Delta L) = \mathcal{E}$$

$$E(2\cancel{\pi} r) = \mu_0 n \cancel{\pi} \omega R^2 I_{max} \sin\omega t$$

$$\boxed{E_{out\,(r>R)} = \frac{\mu_0 n \omega R^2 I_{max}}{2r} \sin\omega t} .$$

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$$E_{ins} = \frac{\mu_0 n \omega I_{max}\, r}{2} \sin\omega t$$

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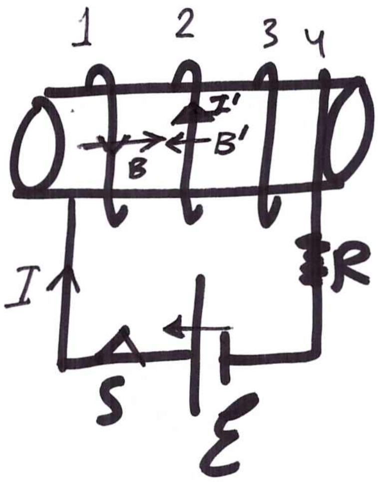
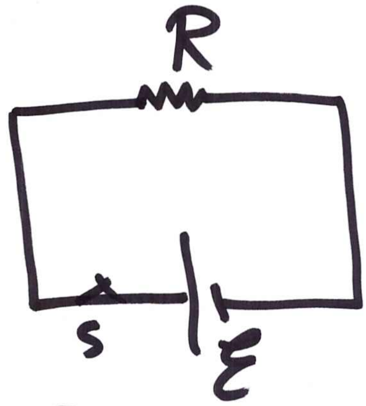
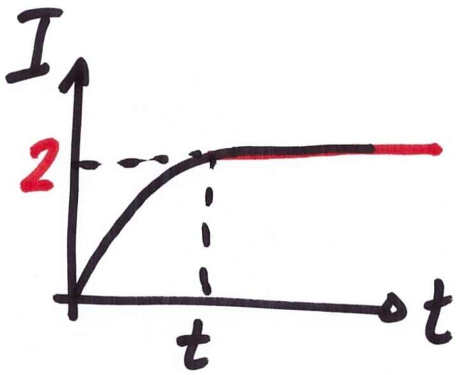
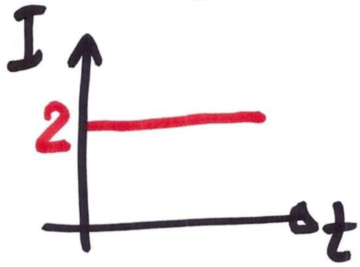
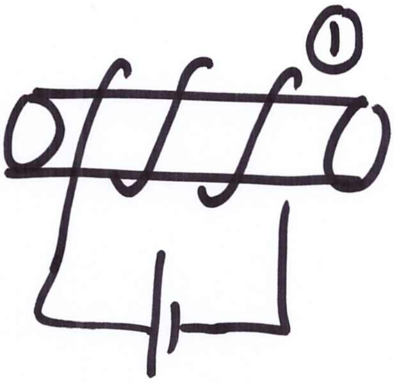
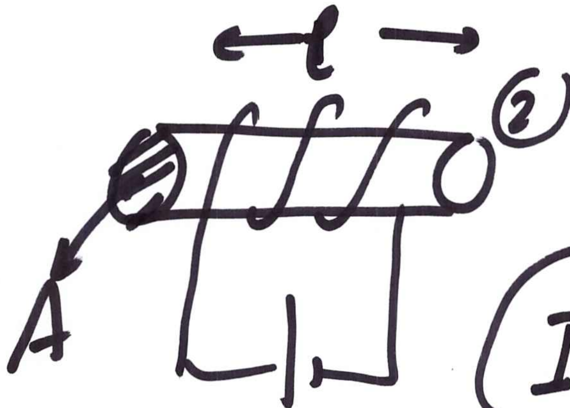








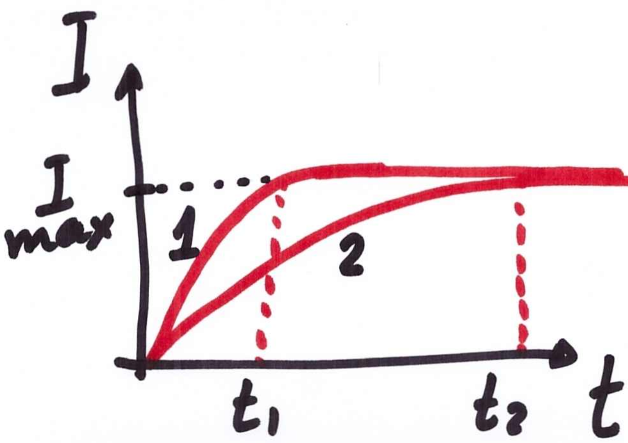




Inductunce

$\Downarrow$

L



$L_2 > L_1$

Self Induction:

$$\mathcal{E}'_{L} = -N\frac{d\phi}{dt} = -N\frac{\Delta\phi}{dt}$$

general

$\phi = BA\cos\theta$

$d\phi = d(BA\cos\theta)$

$\Delta\phi = \Delta(BA\cos\theta)$

$$\mathcal{E}'_{L} = -L\frac{di}{dt} = -L\frac{\Delta i}{\Delta t}$$

$$i' = \frac{|\mathcal{E}'|}{R}$$

$$L_{general} = \frac{N\Delta\phi}{\Delta i} = \frac{N\phi_B}{i}$$

$$L_{solenoid} = \frac{\mu_0 N^2 A}{\ell} = \mu_0 n^2 \ell A = \mu_0 n^2 \mathrm{Vol}$$

$L \longrightarrow$ Henry (H)

$C = \frac{q}{V}$

$R = \frac{V}{i}$

$L = \frac{N\phi}{i}$

$N = n\ell$

$$U_B = \frac{1}{2}LI^2$$

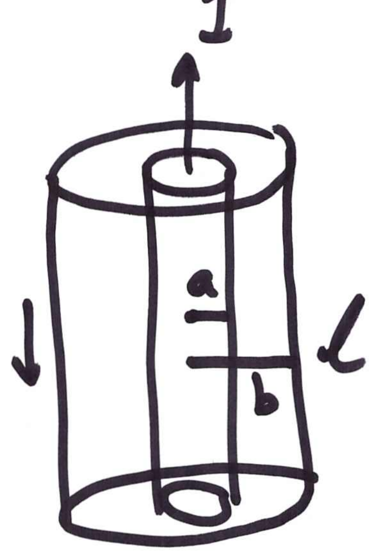
(magnetic energy stored in the magnetic field inside the inductor)

$$u_B = \frac{U}{Vol} = \frac{B^2}{2\mu_0}$$

energy density or "per unit

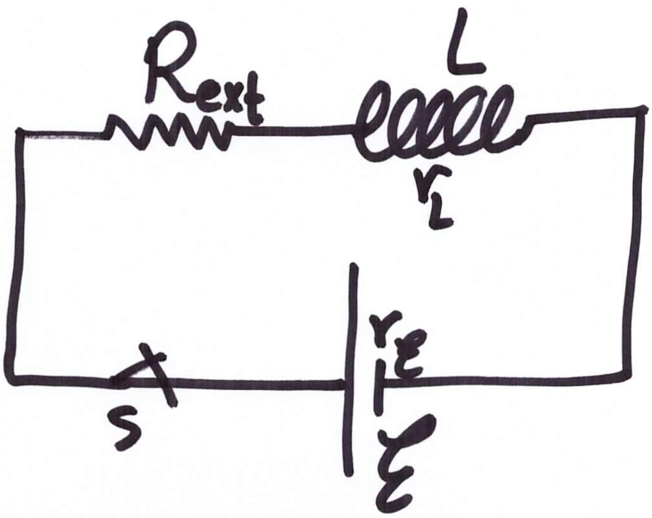
$$u = \frac{1}{2}\epsilon_0 E^2$$

$$\boxed{L_{\text{coaxial cable}} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right)}$$



$$C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$

RL - circuit :



$$R = R_{ext} + r_L + r_\varepsilon$$

$$\varepsilon = iR + L\frac{di}{dt}$$

تشغيل الدارة

	$t=0$	$t=\infty$
i	0	i_{max}
$\frac{di}{dt}$	$\left(\frac{di}{dt}\right)_{max}$	0

$$i_{max} = \frac{\varepsilon}{R}$$

time constant τ

$$\left(\frac{di}{dt}\right)_{max} = \frac{\varepsilon}{L}$$

$$\tau = \frac{L}{R}$$

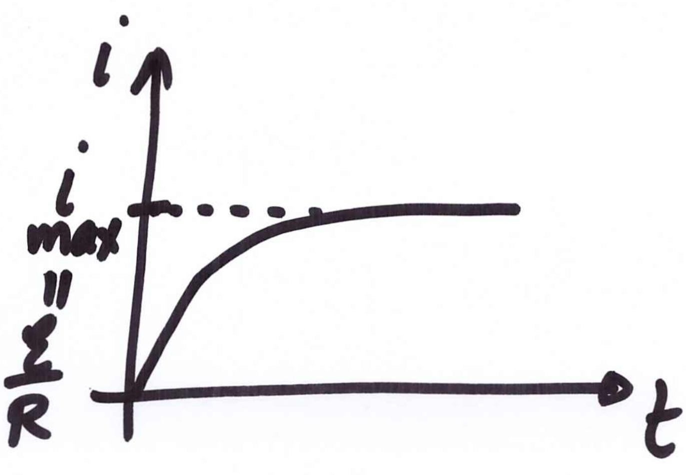
i	di/dt
1) i	1) di/dt
2) $V_R = iR$	2) $V_L = L\frac{di}{dt} + ir_L$
3) $V_\mathcal{E} = \mathcal{E} - ir_\mathcal{E}$	3) $\mathcal{E}' = -L\,di/dt$
4) $P_R = i^2R$	4) $Power_{(L)} = iL\frac{di}{dt}$
5) $P_\mathcal{E} = i\mathcal{E}$	
6) $U_L = \frac{1}{2}Li^2$	

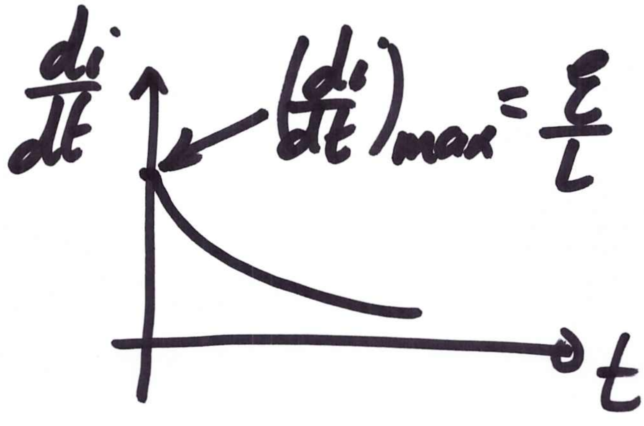
$$\mathcal{E} = iR + L\frac{di}{dt}$$

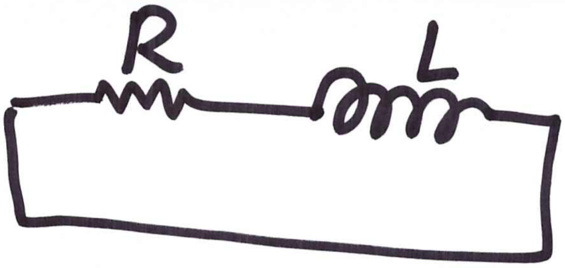
$$i = \frac{1}{2}i_{max}$$

$$i = i_{max}\left(1 - e^{-Rt/L}\right) \qquad \tau = \frac{L}{R}$$

$$\frac{di}{dt} = \left(\frac{di}{dt}\right)_{max} e^{-Rt/L}$$





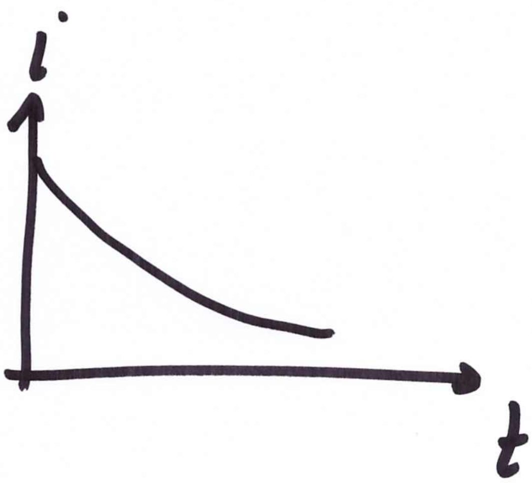


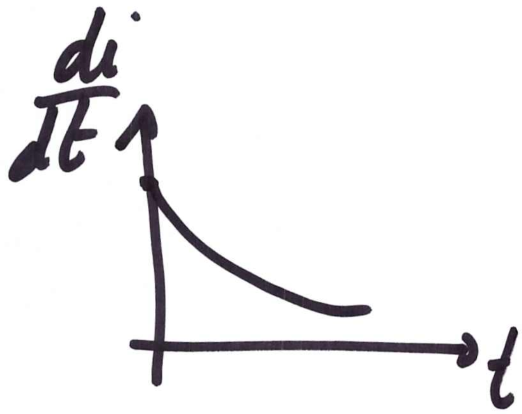
$$0 = iR + L\frac{di}{dt}$$

$$i_{max} = \frac{\mathcal{E}}{R} \; , \; \left(\frac{di}{dt}\right)_{max} = \frac{\mathcal{E}}{L} \; , \; \tau = \frac{L}{R}$$

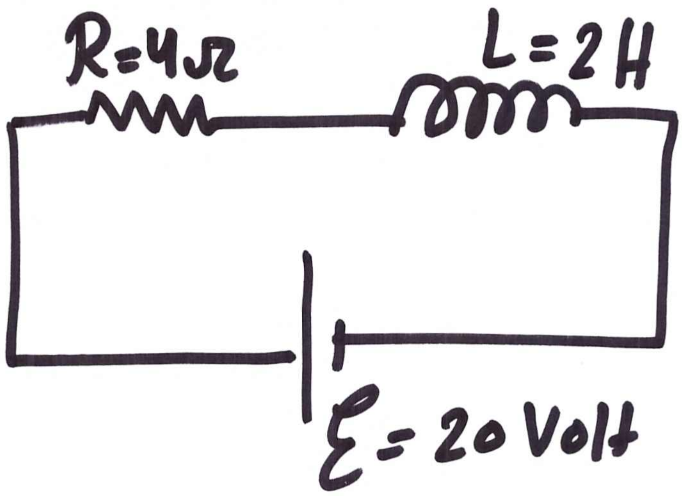
$$i = i_{max}\, e^{-t/\tau}$$

$$\left(\frac{di}{dt}\right)_{\cancel{max}} = -\left(\frac{di}{dt}\right)_{max} e^{-t/\tau}$$





Ex:



Find: 1) i_{max} , $\left(\frac{di}{dt}\right)_{max}$, τ .

2) if $i = \frac{1}{5} i_{max}$, Find :

a) $\frac{di}{dt}$ b) Voltage accross the inductor

c) induced emf ($\mathcal{E}'$) d) Power in L .

3) if $\frac{di}{dt}$ is equal to 40% of its max. value, what is :

a) current (i) b) Voltage accross (R)

c) Power in R d) " " ($\mathcal{E}$)

e) " " $\mathcal{E}$ f) energy in the inductor

4) at $t = 2$-sec, find:

a) current b) change rate of current $\left(\frac{di}{dt}\right)$

5) find Current (i) and $\frac{di}{dt}$ after 3-time constant

6) find time needed to reach to $\frac{1}{2}$ max. Current

7) " " " " " " $\frac{1}{10}$ " $\frac{di}{dt}$.

Sol: $R = 4\Omega$, $L = 2H$, $\mathcal{E} = 20$ Volt

① $\tau = \frac{L}{R} = \frac{2}{4} = 0.5$ sec

$$i_{max} = \frac{\mathcal{E}}{R} = \frac{20}{4} = 5A$$

$$\left(\frac{di}{dt}\right)_{max} = \frac{\mathcal{E}}{L} = \frac{20}{2} = 10 \, A/s$$

2) $\boxed{i = \frac{1}{5} i_{max} = \frac{1}{5} * 5 = 1A}$ $\left(\frac{1}{5} = 20\%\right)$

a) $\mathcal{E} = iR + L\frac{di}{dt}$

$20 = 1 * 4 + 2\frac{di}{dt}$

$\boxed{\frac{di}{dt} = 8 A/s}$

b) $V_L = L\frac{di}{dt} + \cancel{i r_L}$

$= 2 * 8$

$= 16$ Volt

c) $\mathcal{E}' = -L\frac{di}{dt} = -16V$

d) $P_L = i L\frac{di}{dt}$

$= 1 * 2 * 8$

$= 16$ watt

3) $\frac{di}{dt} = \frac{40}{100}\left(\frac{di}{dt}\right)_{max} = \frac{40}{100} * 10 = 4 A/s$.

a) $\mathcal{E} = iR + L\frac{di}{dt}$

$20 = i * 4 + 2 * 4$

$\boxed{i = 3A}$

b) $V_R = iR$

$= 3 * 4$

$= 12$ Volt

c) $P_R = i^2 R$

$= 9 * 4 = 36$ Volt

d $V_{\mathcal{E}} = \mathcal{E} - \cancel{ir}$

$= 20$ Volt

e $P_{\mathcal{E}} = i\mathcal{E} = 3 * 20 = 60$ watt

f $U_L = \frac{1}{2} L i^2$

$= \frac{1}{2} * 2 * (3)^2 = 9$ J.

4 $\underline{\underline{t=2}}$ $\tau = 0.5$ sec

$i = i_{max}(1 - e^{-t/\tau})$

$= 5(1 - e^{-2/0.5}) = 4.9$ A

$\frac{di}{dt} = \left(\frac{di}{dt}\right)_{max} e^{-t/\tau}$

$= 10 \; e^{-2/0.5} = 0.18$ A/s

5 $t = 3\tau$

$i = 5(1 - e^{-3\cancel{\tau}/\cancel{\tau}}) = 4.75\,A$.

$\frac{di}{dt} = 10\, e^{-3\cancel{\tau}/\cancel{\tau}} = 0.5\ A/s$.

6 $i = i_{max}(1 - e^{-t/\tau})$

$\frac{1}{2}\cancel{i_{max}} = \cancel{i_{max}}(1 - e^{-t/\tau})$

$\frac{1}{2} = 1 - e^{-t/0.5}$

$\cancel{-}\frac{1}{2} = \cancel{-}e^{-t/0.5} \Rightarrow \ln \Rightarrow \frac{-t}{0.5} = \ln 0.5 = -0.693$

$\boxed{t = 0.35\ sec}$

7 $\frac{di}{dt} = \left(\frac{di}{dt}\right)_{max} e^{-t/\tau}$

$0.1\cancel{\left(\frac{di}{dt}\right)_{max}} = \cancel{\left(\frac{di}{dt}\right)_{max}} e^{-t/\tau} \Rightarrow 0.1 = e^{-t/\tau} \Rightarrow \ln$

$\ln 0.1 = \frac{-t}{0.5} \Rightarrow t = 1.15\ sec$

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